Construct LR(1) States

$$S_0$$

1 $$\langle S \rangle ::= a \langle A \rangle \langle B \rangle e$$
2 $$\langle A \rangle ::= \langle A \rangle b c$$
3 $$\langle A \rangle ::= b$$
4 $$\langle B \rangle ::= d$$
Construct LR(1) States

1  \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
2  \( \langle A \rangle ::= \langle A \rangle b c \)
3  \( \langle A \rangle ::= b \)
4  \( \langle B \rangle ::= d \)

\[ S' ::= \star S$, $\]
\[ S ::= \star \langle A \rangle \langle B \rangle e$, $\]
\[ S ::= a \langle A \rangle \langle B \rangle e, $\]
\[ \langle A \rangle ::= \star \langle A \rangle b c, d \]
\[ \langle A \rangle ::= \star \langle A \rangle b c, b \]
\[ \langle A \rangle ::= \star b, d \]
\[ \langle A \rangle ::= \star b, b \]
\[ S' ::= S \star S, $\]

S0

S1

S2

a

b
Construct LR(1) States

1. \( \langle S \rangle ::= \langle A \rangle \langle B \rangle \ e \)
2. \( \langle A \rangle ::= \langle A \rangle \ b \ c \)
3. \( \langle A \rangle ::= b \)
4. \( \langle B \rangle ::= d \)

\[
\begin{array}{c|c}
S0 & [ S' ::= \bullet S$, $] \\
\ [ S ::= \bullet a \langle A \rangle \langle B \rangle e,$ $] \\
S1 & [ S' ::= S$\bullet$, $] \\
S & [ S ::= \bullet a \langle A \rangle \langle B \rangle e,$ $] \\
S2 & [ S ::= a \bullet \langle A \rangle \langle B \rangle e,$ $] \\
S3 & [ \langle A \rangle ::= \bullet \langle A \rangle bc,d ] \\
S4 & [ \langle A \rangle ::= \bullet \langle A \rangle bc,b ] \\
\end{array}
\]
Construct LR(1) States

1. \( \langle S \rangle ::= a \: \langle A \rangle \: \langle B \rangle \: e \)
2. \( \langle A \rangle ::= \langle A \rangle \: b \: c \)
3. \( \langle A \rangle ::= b \)
4. \( \langle B \rangle ::= d \)
Construct LR(1) States

1. $\langle S \rangle ::= a \langle A \rangle \langle B \rangle e$
2. $\langle A \rangle ::= \langle A \rangle b c$
3. $\langle A \rangle ::= b$
4. $\langle B \rangle ::= d$
Construct LR(1) States

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
2. \( \langle A \rangle ::= \langle A \rangle b c \)
3. \( \langle A \rangle ::= b \)
4. \( \langle B \rangle ::= d \)
Construct LR(1) States

1. \(\langle S\rangle ::= a \langle A\rangle \langle B\rangle e\)
2. \(\langle A\rangle ::= \langle A\rangle b c\)
3. \(\langle A\rangle ::= b\)
4. \(\langle B\rangle ::= d\)
Construct LR(1) States

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle \ e \)
2. \( \langle A \rangle ::= \langle A \rangle \ b \ c \)
3. \( \langle A \rangle ::= b \)
4. \( \langle B \rangle ::= d \)
Construct LR(1) Parse Table

\[ S'::= S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]

1. \( \langle S \rangle::= a \langle A \rangle \langle B \rangle e \)
2. \( \langle A \rangle::= a \langle A \rangle \langle B \rangle e \)
3. \( \langle A \rangle::= b \)
4. \( \langle B \rangle::= d \)

\[ S \]
\[ S'::= S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]
\[ S::= a \langle A \rangle \langle B \rangle e, S \]

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<th>b</th>
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<th>d</th>
<th>e</th>
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</table>

s i -> shift to State i;
g i -> goto State i;
r i -> reduce with Rule i;
Construct LR(1) Parse Table

<table>
<thead>
<tr>
<th></th>
<th>$S$ ::= $A$ $B$ $e$</th>
<th>$A$</th>
<th>$B$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$S$ ::= $a$ $A$ $B$ $e$</td>
<td>$a$</td>
<td></td>
<td>$S0$</td>
</tr>
<tr>
<td>2</td>
<td>$A$ ::= $A$ $b$ $c$</td>
<td></td>
<td>$b$</td>
<td>$S2$</td>
</tr>
<tr>
<td>3</td>
<td>$A$ ::= $b$</td>
<td></td>
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<td>$S1$</td>
</tr>
<tr>
<td>4</td>
<td>$B$ ::= $d$</td>
<td></td>
<td></td>
<td>$S3$</td>
</tr>
</tbody>
</table>

- $s_i \rightarrow$ shift to State $i$;
- $g_i \rightarrow$ goto State $i$;
- $r_i \rightarrow$ reduce with Rule $i$;
Construct LR(1) Parse Table

<table>
<thead>
<tr>
<th>State</th>
<th>Action</th>
<th>Input</th>
<th>Look-Ahead</th>
<th>Action/State</th>
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<tr>
<td>S0</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>S2</td>
</tr>
<tr>
<td>S1</td>
<td>b</td>
<td>b</td>
<td>d</td>
<td>S3</td>
</tr>
<tr>
<td>S2</td>
<td>c</td>
<td>c</td>
<td>d</td>
<td>S4</td>
</tr>
<tr>
<td>S3</td>
<td>d</td>
<td>d</td>
<td>e</td>
<td>S5</td>
</tr>
<tr>
<td>S4</td>
<td>e</td>
<td>e</td>
<td>$</td>
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</tr>
<tr>
<td>S5</td>
<td>$</td>
<td>A</td>
<td>B</td>
<td>S7</td>
</tr>
<tr>
<td>S6</td>
<td>S</td>
<td>A</td>
<td>B</td>
<td>S8</td>
</tr>
<tr>
<td>S7</td>
<td>S</td>
<td>A</td>
<td>B</td>
<td>S9</td>
</tr>
</tbody>
</table>

s i − > shift to State i;
g i − > goto State i;
r i − > reduce with Rule i;
Construct LR(1) Parse Table

**S0**

- \([ S' ::= S$ \), $\]
- \([ S ::= a \langle A \rangle \langle B \rangle e, $ \]

**S1**

- \([ S' ::= S \cdot S$, $] \)

**S2**

- \([ S ::= a \cdot \langle A \rangle \langle B \rangle e, $ \]
- \([ \langle A \rangle ::= \cdot \langle A \rangle b c, d \]
- \([ \langle A \rangle ::= \cdot \langle A \rangle b c, b \]
- \([ \langle A \rangle ::= \cdot b, d \]
- \([ \langle A \rangle ::= \cdot b, b \]

**S3**

- \([ \langle A \rangle ::= \cdot b, b \]
- \([ \langle A \rangle ::= \cdot b, c d \]

**S4**

- \([ S ::= a \langle A \rangle \cdot \langle B \rangle e, $ \]
- \([ \langle A \rangle ::= \cdot \langle A \rangle b c, d \]
- \([ \langle B \rangle ::= \cdot d, e \]

**S5**

- \([ \langle A \rangle ::= \cdot \langle A \rangle b c, d \]
- \([ \langle A \rangle ::= \cdot d, e \]

**S6**

- \([ \langle A \rangle ::= \cdot \langle A \rangle b c, b \]

**S7**

- \([ \langle A \rangle ::= \cdot \langle A \rangle b c, c \]

**S8**

- \([ \langle B \rangle ::= \cdot d, e \]

**S9**

- \([ \langle A \rangle ::= \cdot \langle A \rangle b, b \]

**Transition Table**

<table>
<thead>
<tr>
<th></th>
<th>( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e )</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>$</th>
<th>A</th>
<th>B</th>
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<tbody>
<tr>
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<td>( \langle A \rangle ::= a \langle A \rangle \langle B \rangle e )</td>
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<td></td>
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<td></td>
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</tr>
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<td>( \langle A \rangle ::= a \langle A \rangle \langle B \rangle e )</td>
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- si – → shift to State i;
- gi – → goto State i;
- ri – → reduce with Rule i;
Construct LR(1) Parse Table

<table>
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<tr>
<th></th>
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s i − > shift to State i;
g i − > goto State i;
r i − > reduce with Rule i;
Construct LR(1) Parse Table

\[ S' ::= S \]
\[ S ::= a \langle A \rangle \langle B \rangle e, S \]
\[ S ::= \bullet a \langle A \rangle \langle B \rangle e, S \]
\[ S' ::= S \bullet \$, S \]

1. \[ \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \]
2. \[ \langle A \rangle ::= \langle A \rangle b c \]
3. \[ \langle A \rangle ::= b \]
4. \[ \langle B \rangle ::= d \]

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s_i \rightarrow \text{shift to State } i;
g_i \rightarrow \text{goto State } i;
r_i \rightarrow \text{reduce with Rule } i;
Construct LR(1) Parse Table

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<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
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<td>$</td>
<td>( \langle A \rangle )</td>
<td>( \langle B \rangle )</td>
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<td>3</td>
<td>( \langle A \rangle ::= b )</td>
<td>( \text{acc} )</td>
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<td>4</td>
<td>( \langle B \rangle ::= d )</td>
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</table>

s \( i \) \( \rightarrow \) shift to State \( i \);
g \( i \) \( \rightarrow \) goto State \( i \);
r \( i \) \( \rightarrow \) reduce with Rule \( i \);
Construct LR(1) Parse Table

![Diagram of LR(1) Parse Table]

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<th>B</th>
<th>S</th>
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<td>$S::=a \langle A \rangle \langle B \rangle e,$</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
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s i − > shift to State i;
g i − > goto State i;
r i − > reduce with Rule i;
Construct LR(1) Parse Table

<table>
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<th>Rule</th>
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<th>State</th>
<th>Symbol</th>
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$S0 \rightarrow \epsilon$

$S1 \rightarrow S \epsilon$

$s \rightarrow >$ shift to State $i$;
$g \rightarrow >$ goto State $i$;
$r \rightarrow >$ reduce with Rule $i$;
Construct LR(1) Parse Table

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
   - \( a \quad b \quad c \quad d \quad e \quad \$ \)
   - \( A \quad B \quad S \)
   - S0: s2
   - S1: acc
   - S2: s3
   - S3: r3  r3
   - S4: s5  s9  g7
   - S5: s6
   - S6: r2
   - S7: s8
   - S8: r1
   - S9: r4

s i → shift to State i;
g i → goto State i;
r i → reduce with Rule i;
LR(1) Pushdown Automata (PDA)

- Summary
  - States - canonical collection of LR(1) items
  - Edges - goto transitions
  - Reducing a handle = returning to a prior state
  - Need a stack to save previous states and symbols
Example

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
2. \( \langle A \rangle ::= \langle A \rangle \ b \ c \)
3. \( \langle A \rangle ::= b \)
4. \( \langle B \rangle ::= d \)

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- INV S0

Remaining INPUT: abbcde
LR(1) PDA - Putting It All Together

Example

1. \langle S \rangle ::= a \langle A \rangle \langle B \rangle \epsilon
2. \langle A \rangle ::= \langle A \rangle b c
3. \langle A \rangle ::= b
4. \langle B \rangle ::= d

Remaining INPUT: bbcde

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Example

1. $S \rightarrow aA\langle B\rangle e$
2. $A \rightarrow \langle A \rangle b c$
3. $A \rightarrow b$
4. $B \rightarrow d$

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Remaining INPUT: bcde
Example

1  \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
2  \( \langle A \rangle ::= \langle A \rangle \ b \ c \)
3  \( \langle A \rangle ::= b \)
4  \( \langle B \rangle ::= d \)

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\[ \begin{array}{|c|c|c|c|c|c|c|c|c|}
  \hline
  a & S2 & \hline
  INV & S0 & \hline
  \end{array} \]

Remaining INPUT: bcde
LR(1) PDA - Putting It All Together

Example

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
2. \( \langle A \rangle ::= \langle A \rangle b c \)
3. \( \langle A \rangle ::= b \)
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Remaining INPUT: bcde
LR(1) PDA - Putting It All Together

Example

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
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Remaining INPUT: bcde
Example

1  \langle S \rangle ::= a \langle A \rangle \langle B \rangle e
2  \langle A \rangle ::= \langle A \rangle b c
3  \langle A \rangle ::= b
4  \langle B \rangle ::= d

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Remaining INPUT: cde
LR(1) PDA - Putting It All Together

Example

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
2. \( \langle A \rangle ::= \langle A \rangle b \ c \)
3. \( \langle A \rangle ::= b \)
4. \( \langle B \rangle ::= d \)

```
\begin{array}{|c|c|c|c|c|c|c|}
\hline
a & b & c & d & e & $ & \text{A} \ 	ext{B} \ 	ext{S} \\
\hline
S0 & s2 & & & & & g1 \\
S1 & & & & & \text{acc} & \\
S2 & & s3 & & & & g4 \\
S3 & r3 & r3 & & & & \\
S4 & s5 & s9 & & & g7 \\
S5 & & s6 & & & & \\
S6 & & & & r2 & & \\
S7 & & & & s8 & & \\
S8 & & & & r1 & & \\
S9 & & & & & r4 & \\
\hline
\end{array}
```

- c S6
- b S5
- A S4
- a S2
- INV S0

Remaining INPUT: de
Example

1. \( \langle S \rangle ::= a \langle A \rangle \langle B \rangle e \)
2. \( \langle A \rangle ::= \langle A \rangle b c \)
3. \( \langle A \rangle ::= b \)
4. \( \langle B \rangle ::= d \)

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Remaining INPUT: de
LR(1) PDA - Putting It All Together

Example

1. $\langle S \rangle ::= a \langle A \rangle \langle B \rangle e$
2. $\langle A \rangle ::= \langle A \rangle b c$
3. $\langle A \rangle ::= b$
4. $\langle B \rangle ::= d$

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Remaining INPUT: de
Example

1. $\langle S \rangle ::= a \langle A \rangle \langle B \rangle e$
2. $\langle A \rangle ::= \langle A \rangle b c$
3. $\langle A \rangle ::= b$
4. $\langle B \rangle ::= d$

|   | a | b | c | d | e | $|$ | A | B | S |
|---|---|---|---|---|---|---|---|---|---|
| S0 | s2 |   |   |   |   |   | $|$ |   | g1 |
| S1 |   | acc |   |   |   |   |   |   |   |
| S2 | s3 |   |   |   |   |   |   | g4 |   |
| S3 | r3 | r3 |   |   |   |   | g4 |   |   |
| S4 | s5 | s9 |   |   |   |   | g7 |   |   |
| S5 |   | s6 |   |   |   |   |   |   |   |
| S6 |   | r2 |   |   |   |   |   |   |   |
| S7 |   | s8 |   |   |   |   |   |   |   |
| S8 |   |   |   |   |   |   |   | r1 |   |
| S9 |   |   |   |   |   |   |   |   | r4 |

A S4
a S2
INV S0

Remaining INPUT: de
Example

1  ▶  \(\langle S\rangle ::= a\langle A\rangle\langle B\rangle e\)
2  \(\langle A\rangle ::= \langle A\rangle b c\)
3  \(\langle A\rangle ::= b\)
4  \(\langle B\rangle ::= d\)

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\begin{array}{|c|c|c|c|c|c|c|}
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\text{A} & \text{B} & \text{S} \\
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\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{$\}$} & \text{g1} \\
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\text{S1} & \text{acc} & \text{g4} \\
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\text{S8} & \text{r1} \\
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\text{S9} & \text{r4} \\
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\end{array}
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Remaining INPUT: e
### Example

1. $\langle S \rangle ::= a \langle A \rangle \langle B \rangle e$
2. $\langle A \rangle ::= \langle A \rangle b c$
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Remaining INPUT: e

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LR(1) PDA - Putting It All Together

Example

1. $\langle S \rangle ::= a \langle A \rangle \langle B \rangle e$
2. $\langle A \rangle ::= \langle A \rangle b c$
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Remaining INPUT: e
Example

1 | \langle S\rangle ::= a \langle A \rangle \langle B \rangle e
2 | \langle A \rangle ::= \langle A \rangle b c
3 | \langle A \rangle ::= b
4 | \langle B \rangle ::= d

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Remaining INPUT: e
Example

1. $S ::= aA(B)e$
2. $A ::= a b c$
3. $A ::= b$
4. $B ::= d$

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A
  / \  /
 B  A  B
  /  /  /
 a  b  b  c  d  e

Remaining INPUT:
```
Example

1. \( S \) ::= a \langle A \rangle \langle B \rangle e
2. \( A \) ::= \langle A \rangle b c
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 Remaining INPUT:
LR(1) PDA - Putting It All Together

▶ Example

1. $\langle S \rangle ::= a \langle A \rangle \langle B \rangle e$
2. $\langle A \rangle ::= \langle A \rangle b c$
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Remaining INPUT:

```
a b b c d e
```
LR(1) PDA - Putting It All Together

Example

1. $S ::= a A B e$
2. $A ::= A b c$
3. $A ::= b$
4. $B ::= d$

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Remaining INPUT: