CS510 **Test 1** **Oct. 31, 2018**

Instructions: Do all your work in the blue examination booklets (if you do not have one, or need more space, you may use your own normal-size paper.

Write answers IN THE GIVEN ORDER, though you may work on them in any order.

You may use one page of prepared notes (both sides, but normal size writing); ALSO the test is CLOSED BOOK.

Show ALL your work. You will get little or no credit for an unexplained answer. The value of each question appears in parentheses, 85 points in all. You have 80 minutes.

Although you are doing this AS A TAKEHOME, I ask you TO ACT AS IF YOU WERE IN A SUPERVISED CLASSROOM:

(i) observe the time limit;

(ii) no books or notes or other help (internet, friends, classmates) beyond your one prepared page.

When you finish please write "I have followed the rules for this exam" and then sign your name after this pledge.

1. Consider the linear system

$$A' = (A|\underline{b}) = \left(\begin{array}{cc|c} -2 & 5 & 3 \\ 1 & 2 & 3 \end{array} \right).$$

   (a) (9 pts) Find A^{-1} using Gauss-Jordan elimination with NO row interchanges. Then use it to find the solution to $A\underline{x} = \underline{b}$. Explain all your steps.

   (b) (10 pts) Find the LUP factorization of A using Gaussian elimination with partial pivoting, then back-solving for the solution. Again, explain what you are doing.

   (c) (7 pts) Repeat the factorization and solution but now using scaled-partial pivoting, and explaining your steps. Is this solution unique? [prove your assertion].

   (d) (9 pts) In this part, $A' = (A|\underline{b})$ is now a system of n linear equations in n unknowns where the coefficients satisfy (i) $a_{ij} = 0$ if $i+j \leq n$ and (ii) $a_{ij} \neq 0$ if $i+j = n+1$. Argue that the system has a unique solution. How much work (the number of $*$ and / steps used) is needed to find it? Explain how you got your answer.

2. We seek the roots of $f(x) = e^{x-1} - x$.

   (a) (4 pts) Show that $w = 1$ is the only root of f.

   (b) (6 pts) Will Newton's method converge if you start at a value P_0 that is close enough to 1? Explain. If "YES", at what rate?

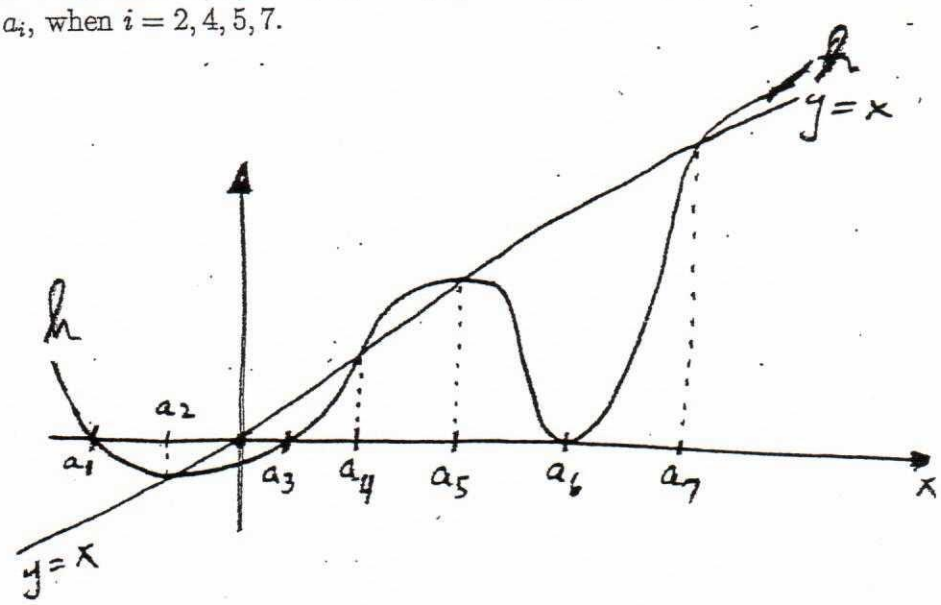
   (c) (5 pts) Can you use bisection to find the root? Explain.

   (d) *(5 pts) Describe what will happen if you use FPI on $g(x) = e^{x-1}$?

   (e) (10 pts) Get three regula-falsi approximations to the root of $f(x) = x^3 - 4$ starting with the interval $(a, b) = (1, 2)$ and then accellerate the last one.

(OVER)

3. The following graph shows a function $h(x)$ and the line $y = x$. As the graph suggests, $h''(x) > 0, a_2 < x < a_4$. Also $h(a_1) = h(a_3) = h(a_6) = h'(a_2) = h'(a_6) = 0 > h'(a_5) > -1$ and $h(a_i) = a_i$, when $i = 2, 4, 5, 7$.



(a) (15 pts) We will do fixed point iteration on h $[P_{n+1} = h(P_n)]$ starting at $P_0 = a_4 + \epsilon$, where $\epsilon > 0$ is <u>much</u> smaller than $a_5 - a_4$. Will it converge? If YES, to what value, and at what rate? Explain. If NO, what happens? Explain. Repeat for $P_0 = a_4 - \epsilon$, where $\epsilon > 0$ is much smaller than $a_4 - a_3$. Can FPI reach $x = a_4$? Explain.

(b) (5 pts) Repeat (a), now starting at $P_0 = a_6$.

(c) (5 pts) Now we will use Newtons method on h starting at $p_0 = a_3 + \epsilon$, where $\epsilon > 0$ is <u>much</u> smaller than $a_4 - a_3$. Will it converge? If YES, to what value, and at what rate? Explain. If NO, what happens? Explain.