CS323  Test 1  Oct. 28, 2019

Instructions:

• Do all your work in the blue examination booklets.
• Write answers IN THE GIVEN ORDER, though you may work on them in any order.
• You may use one page of prepared notes.
• Show ALL your work. You will get little or no credit for an unexplained answer.
• The value of each question appears in parentheses, 78 points in all; you have 75 minutes.

1. \( f(x) = (x^2 - 3) \) has roots at \( w_1 = \sqrt{3} \) and \( w_2 = -w_1 \). We want a good approximation of \( w_1 \).
   (a) (8 pts) Sketch \( f \) on the interval \([-3,3]\). Then DO two steps of Newton’s FPI method for \( w_1 \), starting from \( P_0 = 3 \) (i.e., find \( P_1 \) and \( P_2 \)).
   (b) (7 pts) Will Newton’s method converge to \( w_1 \)? If “YES”, say why, and at what rate?
   (c) (8 pts) Now do one step of the secant method starting from \( P_0 \) and \( P_1 \), the initial Newton approximations. Which is better, the Newton \( P_2 \), or the secant \( P_2 \) approximation (explain your answer)? Will the secant method converge here? If so at what rate?
   (d) (7 pts) Now starting from the same \( P_0 \) used in Newtons method, do two steps of the chord method WITH \( m = 10 \). Will the cord iterations converge to a root?
   (e) (10 pts) Now we will use Aitkin’s method to accelerate the chord method iterations, starting with \( P_2 \). Once you have accelerated \( P_2 \), carefully describe what \textit{would be} the next two steps in following the Aitkin accelerations of the chord method.
   (f) (8 pts) Analogous to the question above, you are now asked to apply \textit{Steffanson accelleration} to the chord method iterations, starting with \( P_2 \), the second chord approximation. Carefully describe how to accelerate \( P_2 \) and then, describe \textit{what would be} the next two steps, once the chord \( P_2 \) was accellerated.

2. (15 pts) Consider the linear system
   
   \[
   A' = (A|b) = \begin{pmatrix}
   3 & 2 & | & 5 \\
   -4 & 1 & | & -3
   \end{pmatrix}.
   \]

   (a) Solve \( Ax = b \) using Gaussian elimination with partial pivoting. Make it explicit what you are doing in each step, and why. Would scaled-partial pivoting take any different steps?
   (b) Now find \( A^{-1} \) and use it to obtain the solution to our system, explaining your steps. Which of the two methods used more multiply and divide operations? Explain.