Instructions: Do all your work in the blue examination booklets. Answer the questions IN THE GIVEN ORDER, though you could work on them in any order. The test is open book, but all work must be your own. Show ALL your work. You will get little or no credit for an unexplained answer. The value of each question appears in parentheses. Use this as a guide in allocating your time. An asterisk (*) denotes a more challenging one.

1. The number $e = 2.7182818284...$ is the base of natural logarithms. We are working in a $k = 3$ digit computer with rounding.
   (a) (5 pts.) What are the two $k$ digit floating point numbers that are closest to $e$?
   (b) (5 pts) Show how the $k$ digit computer will compute $4e$.
   (c) (10 pts) Let $x^*$ be a $k$ digit floating point number that approximates $e$ with relative error less that $10^{-3}$. What is the smallest $x^*$ can be? What is the largest?

2. Given the following data
   
   $A = \begin{pmatrix} 1 & 2 \\ -2 & 7 \end{pmatrix}, \quad D = \begin{pmatrix} -2 & 7 \\ -1/2 & 11/2 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad p = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   (a) (10 pts) Solve the system $Ax = b$ using Gaussian elimination and backsolving no partial-pivoting, no scaled partial pivoting. Describe each row operation you do and and explain the backsolving.
   (b) (5 pts) It is claimed that $D$ is the LU factorization of $A(p)$, in compact form. Write down the factors $L$ and $U$ and decide if the claim is true, explaining your reasoning
   (c) (10 pts) Find $A^{-1}$ using the Gauss-Jordan method (no pivoting strategy). Explain each row operation you do. Finally, use your answer to solve $A\underline{x} = b$. Explain.
   (d) (10 pts) If you were starting Gaussian elimination on a linear system with the following coefficient matrix
   
   $\begin{pmatrix} 1 & 2 & -3 \\ -2 & 7 & -4 \\ -1 & -2 & 2 \end{pmatrix}$

   what is the pivot row for column 1 according to partial pivoting? What is the pivot row according to scaled partial pivoting? Explain.

3. Let $g(x) = x^2 - 2$ and $h(x) = x^3 - 2$.
   (a) (5 pts) Graph $g(x)$ showing all roots and fixed points. Now find the value of all fixed points. Explain.
   (b) (5 pts) Do two FPI steps on $g(x)$ starting from $P_0 = 1$. Will the iterations converge? If so to what?
   (c) (5 pts) We would like to use bisection to find the positive root of $h(x)$ starting with the interval $(0, 4)$. Will it converge? Explain. How many bisection steps are needed to guarantee an error of less than .001?
   (d) (5 pts, *10 pts) Do one step of Newton’s method to approximate $w$, the positive root of $h(x)$, starting from $P_0 = 1$. Investigate the convergence of Newton’s method to $w$, starting at $P_0$. 