Hand in solutions to problems marked with a (*) in class next Wed.
The other questions are for your own practice and benefit, but won’t be graded.
Question 6 is changed from the earlier version.

1. (*) Let \( f(x) = x^2 \) on \([0, 1]\). Set up, then solve, the normal equations for \( P_1(x) \), the (continuous) least squares approximation to \( f \) of degree = one, expanded in the monomial basis. Then find \( d_2(f, P_1) = \int_{0}^{1} (f(x) - P_1(x))^2 dx \), the error of the approximation.

2. \( C_1(x) \), the linear Tchebycheff interpolation of \( f \) is \( C_1(x) = x - 1/8 \). Compute \( d_2(f, C_1) \). Also compute \( d_{\infty}(f, C_1) = \max(|f(x) - C_1(x)|, 0 \leq x \leq 1) \) and \( d_{\infty}(f, P_1) \). What do these calculations show?

3. Now repeat using the basis \( \phi_0(x) = 1 \) and \( \phi_1(x) = 2x - 1 \). Check that the two expressions for \( P_1 \) are equivalent. Finally, show how to use the answer in 1) to find \( P_1 \) in the second basis without setting up and solving normal equations.

4. Repeat for \( g(x) = e^x \) on \([0, 1]\).

5. (*) Take \( x_i = i/5 \), \( i = 0, \ldots, 5 \). Set up and solve the normal equations to find the discrete least squares straight line approximation to \( f(x) = x^2 \) [from (1)], in the monomial basis and then in the basis used in problem 3.

6. (*) Find \( \phi_0, \phi_1, \phi_2 \) the first three orthogonal basis functions for the interval \([-2, 3]\). Then find the coefficient matrix of the normal equations to determine the FIRST (i.e., degree at most ONE, or LINEAR), continuous least squares approximation to \( f \) on this interval in the monomial basis.

7. Let \( f(x) = x^3 \) on \([-2, 3]\). Using the previous results,

   (a) Find \( P_2(x) \), the degree at most two (i.e., quadratic) continuous-least-squares approximation to \( f \), expanded in the monomial basis.

   (b) Now show how to express \( P_2 \) in the orthogonal basis using only the basis functions from 5.

   (c) Finally check the answer in b) by by solving the normal equations for \( P_2 \), expanded in the orthogonal basis.