CS206 **HW2c** **Dec. 6, 2017**

- Hand-in solutions to questions marked by a (*) by Dec. 7, 2017.
- Problems marked by (**) are more challenging, and required for students in the Honors section.
- Problems marked by (+) are NOT required for hand-in and wont be graded, but are probably interesting.
- NOTICE hard question (e) is now NOT REQUIRED!

1. The bridge experiment (model 1) is to choose 13 cards from the deck at random (use equally likely probability).
   (a) What is the probability you get NO aces? What is the probability you get no aces, but you have all the Kings? Whats the conditional probability you got no aces, given that you got all the Kings?
   (b) (*) What is the probability of $A = \{$you get ALL the aces$\}$ but NO Jacks? What is the probability of $B = \{$you get all the aces and the remainimg cards are all spades? What is the conditional probability of B, given A?
   (c) Find the conditional probability of no aces given that you got NO spades.
   (d) (*) What is the probability you get NO spades? What is the probability you get no card higher than 9?
   (e) (**) What is the probability *neither* of the two events in (d) [above], occurs?
2. (*) The bridge experiment (model 2) is a *partition experiment* where (i) 13 cards are chosen at random for the first player (East), (ii) 13 of the remaining cards are chosen at random for the second player (South), (iii) 13 of the remaining cards are chosen at random for the third player (West), and (iv) the remaining cards are given to the last player (North). Repeat question 1 all parts [(a)-(d)] in this model, and, where you are East; i.e., player 1. Is it different if you are West, that is, player 3?
3. (*) There are 10 pairs of shoes in a closet. Five shoes are picked at random. What is the probability there is no "pseudo-pair" in the sample (i.e., one left and one right shoe)? What is the probability that there is no pair?
4. (+)(**) An ordinary deck of 52 cards is shuffled. What is the probability that no aces are adjacent? What is the probability that no spades are adjacent?
5. (+) As above a deck of cards is shuffled. What is the probability that no aces are adjacent or separated by only one non-ace?
6. (+) 4 cards are randomly dealt to each of 13 players.
   (a) Describe the sample space and write down its size.
   (b) What is the probability of $A = \{$each player has one card from each suit$\}$?

(c) What is the probability of $B = \{$each player has all four cards of the same value$\}$?

(d) (*) Compute $P_B(A)$ and $P_A(B)$. Are A and B independent?

(e) What is the probabiility that players one, two, and three have been dealt only aces, kings, or queens. As usual, explain.

(f) (**) What is the probability that *one* player has one card from each suit but that nobody else has cards from *more* than one suit?

7. A fair die is tossed twice. Let $X =$ the sum of the faces, $Y =$ the maximum of the two faces, and $Z = |\text{face } 1 - \text{face } 2|$.

(a) Write down the value of X,Y, Z, and $W = XZ$ for each outcome $w \in S$.

(b) Find $Range(X)$, $Range(Y)$, $Range(Z)$, and $Range(W)$.

(c) Describe the partitions $\mathcal{A}_X$ and $\mathcal{A}_Z$ induced by these random variables.

(d) Find f_X, f_Y, f_Z, and f_W, the frequency functions.

(e) Are the events $A = \{w \in S : X(w) = 7\}$ and $B = \{w \in S : Z(w) = 1\}$ independent?

8. (*) A fair coin is tossed four times. Consider the following random variables on S, the sample space: $X =$ the number of Heads; $Y =$ the length of the longest block of successive Tails (0 if NO Tails); $Z =$ the number of the toss on which the last Tail occurred (0 if NO Tails); $W = \max(X, Y)$; $V = \min(X, Z)$. Use equally likely probability on S.

(a) List the elements of S. For each $w \in S$, write the value of each random variable.

(b) Find the *Range* of each random variable.

(c) Find the frequency function of each random variable and plot it.

(d) Describe the partitions induced by X and W.

(e) Are the events $A = \{w : X(w) = 2\}$ and $B = \{w : Y(w) = 2\}$ independent? Explain.