1. We want to represent each of the following real numbers in a $k = 7$ digit computer: (i) $1.817120593$ (close to $(6)^{1/3}$); (ii) $1.414213562$ (close to $\sqrt{2}$); (iii) $.0013456$; (iv) $101.266$.

(a) Write down the normalized floating point representation using chopping. Compute the error and the relative error.

(b) Repeat using rounding. Which method is more accurate?

2. Suppose you were computing $(6)^{1/3}(\omega)\sqrt{2}$ on a 6-digit computer, where $(\omega)$ stands for one of the arithmetic operators $+,-,\times$ or $\div$. Use (i) and (ii) in problem 1 for the “true” values.

(a) Show what the 6 digit computer would obtain in each of the 4 possibilities for $\omega$ using the chopping convention.

(b) Compute the relative errors of the calculations in a).

(c) Repeat a) and b) using the rounding convention. Is rounding better than chopping?

3. $x'$ is an approximation of $40.214213562$ whose relative error has absolute value less than $5 \times 10^{-5}$. Describe the largest interval in which $x'$ can lie.

4. $a'$, $b'$, and $c'$ are positive integers obtained by chopping the fractional part from $a$, $b$, and $c$, respectively. Find in terms of $a'$, $b'$, and $c'$, the smallest interval containing

(a) $a + b$.

(b) $a \times b$ ($\times$ denotes multiplication).

(c) $(a/b) \times c$.

5. Perform the following computations (i) exactly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. Then compute the relative errors of the final results.

(a) $\frac{3}{4} + \frac{1}{5}$

(b) $\left(\frac{3}{4}\right)\left(\frac{1}{5}\right)$

(c) $\left(\frac{1}{2} - \frac{3}{11}\right)\frac{2}{5} + \frac{3}{20}$

(d) $\left(\frac{1}{2} + \frac{3}{11}\right) + \frac{3}{20}$

(e) Repeat (a) and (c), above, now using four digits. Does this give greater accuracy? Is a $k + 1$ digit computer always more accurate than a $k$ digit one?

6. We want to find the maximum of the function $f(x) = e^{-x} - \cos x$, $x \geq 0$.

(a) On the same graph sketch $e^{-x}$ and $\cos x$, $x > 0$. 

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(b) Argue that $f$ has a global maximum in the interval $(0,2\pi)$. Is this an appropriate starting interval for the bisection method applied to solving $f'(x) = 0$? Carefully explain.

7. You want to find the root of $f(x) = x^3 + 2$.

(a) Starting with the interval $(-2,-1)$ (is this appropriate?) do two steps of bisection.

(b) How many steps are needed to guarantee that the error is less than 0.005?

(c) Now do two regula-falsi steps, as in (a). Which seems better? Explain.

(d) Do two Newton steps starting with $P_0$, the initial bisection from (a).

(e) Do two Secant steps starting with $P_0$ and $P_1$ from (a).

(f) Do three steps of the chord method using $m = 5$.

(g) Now use Aitkin's acceleration to accelerate the last two approximations of the chord method, above.