CS 510 **Homework 1 (Corrected)** **Oct. 12, 2018**

DUE in class Oct. 24, 2018, (or earlier)

Give clear answers to the following questions, explaining what you are doing, and why. You can discuss questions with others, but the written solutions you submit must be entirely your own work, NOT BASED ON ANOTHER STUDENTS WRITEUP, NOR WRITTEN IN COLLABORATION WITH ANYONE ELSE. Before you submit your HW, write the PLEDGE:

- "This write-up is entirely my own work. However I did discuss solutions with ..." (and list your collaborators). Then sign your name.

We wont grade a HW without a pledge. This will be due in approximately 10 days to 2 weeks, the date to be announced this week

1. Suppose you were computing $(-2)^{1/3}(\omega)\sqrt{3}$ on an 8-digit computer, where (ω) stands for one of the arithmetic operators $+, -, *$ or $/$.
   (a) Show what the 8 digit computer would obtain in each of the 4 possibilities for ω using the CHOPPING convention. You can use 1.25992105 for cube root of 2 and 1.73205080 for $\sqrt{3}$
   (b) Now Compute the relative errors of the calculations in a).
   (c) Repeat a) and b) now using using the ROUNDING convention. Does rounding seem better than chopping in this example? EXPLAIN your answer.
   (d) Repeat a) and b), now in $k = 2$ digit arithmetic, and when ω is "-". Have large relative errors occurred here? If so, can you explain why?

2. Recall that if f is n times differentiable, its nth Taylor approximation, expanded about $x = u$, is the polynomial
$$T_n(x) = \sum_{i=0}^{n} a_i(x-u)^i,$$
where $a_0 = f(u)$ and $a_i = f^{(i)}(u)/i!$, ($f^{(i)}$ denotes the ith derivative of f). Taylors theorem says that if $f^{(n+1)}$ is continuous, the error at x is given by
$$f(x) - T_n(x) = \frac{f^{(n+1)}(\theta)(x-u)^{n+1}}{(n+1)!}$$
for some θ between x and u.
   (a) Find the quadratic Taylor approximation to $f(x) = (x)^{1/3}$ expanded about $u = 8/27$. What is its approximation of $f(2/3) = (2/3)^{1/3}$? What is the relative error of this approximation? Now use Taylors Theorem to obtain bounds on $f(2/3) - T_2(2/3)$.
   (b) Find the nth Taylor approximation for $f(x) = \sin x$ expanded about $u = 0$. Show that for any *fixed* x, $T_n(x) \to f(x)$ *as* $n \to \infty$.

3. You want to find the root of $f(x) = x^3 + 4$. (The parts below may involve some tedious calculations for which you might wish to use a small program you write, or Maple, or Matlab, or a calculator, and all these options are permitted. If you dont do it by hand, say how you DID produce your answers to the following.)

   (a) Starting with the interval $(-2, 0)$ (is this appropriate?) do two steps of bisection.

   (b) How many steps are needed to guarantee that the error is less than $5 * 10^{-5}$?

   (c) Do two regula-falsi steps, as in (a). Which seems better, this or bisection? Now accellerate the last approximation, make another r.f. step, and then accellerate IT.

   (d) Do two Newton steps starting with P_0, the initial bisection from (a).

   (e) Do two Secant steps starting with P_0 and P_1 from (a).

4. Let $c > 0$ be a given constant. Study the behaviour of FPI on $g(x) = c + (x - c)^4 + 2(x - c)^3$ starting at $P_0 = c - 1/4$.

5. Investigate the behaviour of the chord method in finding the root of $f(x) = x^3 + 2$, starting with $P_0 = 1$, and using $m = 12$. Repeat for $f(x) = x^3 + 1$, $P_0 = 1$, and $m = 3$.