1. Players A, B, C, and D toss a fair coin in order (i.e., first A then B, then C, then D, then A again, etc.) The first player to get a HEAD wins, and the game is over. What are their respective chances to win? Repeat, now with only three players who toss a pair of fair coins, the first to get something different than HH is the winner, and the game is over.

2. We do the balls and boxes experiment with $r$ balls and $n$ boxes (each ball, labeled with its number - chooses one of the $n$ boxes (each labeled with its number) uniformly at random and is placed in that box. $n > 5$ and each box has capacity $> r$.

Let $N$ denote the number of empty boxes, $Y$ the number of boxes with exactly one ball, and $L_i$ denotes the load on (i.e., number of balls in) box $i$.

(a) Find the probability that $L_1 = k$, $k = 0, 1, \ldots, r$.
(b) Find the expected value for each of these random variables, $N, Y, L_1, L_2$.
(c) Are $N$ and $L_1$ independent? Explain. Repeat for $N$ and $Y$; for $L_1$ and $Y$; for $L_1$ and $L_2$.
(d) If $r = n + 1$, find the probability that $N = 0$. If $r = n - 1$ find the probability that $N = 1$; that $N = 2$.
(e) What is the probability that ALL $L_i = 0, i = 1, \ldots, 5$?

3. 4 cards from a 52 card deck are randomly dealt to each of 13 distinguishable players.

(a) Describe the sample space and write down its size.
(b) What is the probability that one of the players got all the aces? What is the probability that NOBODY got all the aces?
(c) What is the probability that each player has one card from each suit? (Here, and on following parts, carefully explain your counting.)
(d) What is the probability that one player has one card from each suit but that nobody else has cards from more than one suit?

4. (*) Try to show that the expected number of comparisons in Floyd-Rivest needed to get a “good” interval $[L, R]$ is at at most $3 \ast n/2 + o(n)$ [and if you dont believe its true, say why].

5. (*) The final step of the Floyd-Rivest randomized selection algorithm is to sort the (random) set $S = \{a_i \in A : L \leq a_i \leq R\}$ of inputs which lie between sampled items $L$ and $R$, once that set has been verified as “good” (because it must contain the item from $A$ of rank $k$). Show that $P(|S| > 4n^{5/6}) \rightarrow 0$ as $n \rightarrow \infty$. (hints: look at the item $\sigma \in A$ of appropriate rank and study the probability that $L < \sigma$; look at the item $\tau \in A$ of appropriate rank and study the probability that $R > \tau$).