HW1 - Some Solutions

- **1a**: FALSE if $C \supseteq A \cup B$ and $A \neq \emptyset$, e.g. $A = 1, B = 2, C = A \cup B$.
- **1b**(*)&: FALSE if $A \cap B \neq \emptyset$, e.g. $A = 1, 2, B = 2, 3$.
- **1c**: FALSE if $A \subset B$, e.g., $A = 1, B = 1, 2$.
- **1d**(*)&: TRUE. Take $x$ in l.h.s. and argue its in r.h.s., and then vice-versa.
- **1e**: FALSE because the inclusion is STRICT. So if $A_2, \ldots, A_n$ are all subsets of $A_1$, the two sides of the expression will be EQUAL.
- **1f**(*)&: same answer as (f).
- **1i**: FALSE because the inclusion is STRICT, as in 1g. If $A_{n+1}, \ldots, A_{m+n}$ each contain ALL $A_i, i = 1, \ldots, n$ the proper inclusion does not hold.

- **2a**: $A \cap B \cap C$.
- **2b**(*)&: $(A \cap B \cap C)^c$.
- **2c**(*)&: $(A \cap B \cap C^c) \cup (A \cap B \cap C^c) \cup (A^c \cap B \cap C)$.
- **2e**(*)&: $C \cap (A \cup B)$.
- **3a**: $S = \{(t_1, t_2, t_3, t_4) : t_i \in \{\text{HEAD}, \text{TAIL}\} \text{ the outcome of the } i^{th} \text{ toss}\}, |S| = 2^4 = 16$.
- **3c**: $S = \{(f_1, f_2, f_3, f_4, f_5) : f_i \in \{1, 2, 3\} \text{ the floor where person } i \text{ exits}\}, |S| = 3^5 = 243$.
- **3f**: $S = E \cup O$ where $E = \{(f, c) : f \in \{2, 4, 6\} \text{ is the even outcome of the die toss and } c \in \{\text{HEAD}, \text{TAIL}\} \text{ is the outcome of the subsequent coin toss}\}$ and $O = \{(f_1, f_2) : f_1 \in \{1, 3, 5\} \text{ is the odd first die toss and } f_2 \in \{1, 2, 3, 4, 5, 6\} \text{ is the following die toss}\}, |S| = |E| + |O| = 6 + 18 = 24$ possible outcomes.
- **4a**: $A = \{(H, H, H, H), (H, H, T, H), (H, T, H, H), (H, T, T, H)\}$, $B = \{(T, T, T, T), (T, T, T, H), (T, T, H, T), (T, H, T, T), (H, T, T, T)\}$
  union with
  $\{(T, H, H, H), (T, H, T, H), (H, T, H, T), (T, H, H, T), (H, H, T, T)\}$
  $A \cup B$ is all of $A$ and all of $B$ except $(H, T, T, H)$, which is already in $A$, and which is - in fact - $A \cap B$.
- **4d:**
- **4e:**
• **4f**: If you pick a red ball you are done in one step. Otherwise you got blue first and now the quickest you could be done is if you pick red twice in a row, so we now have the two outcomes

\[ r \text{ and } brr \text{ (this is the event } A \text{),} \]

and that’s all that can occur with three or less draws from the box.

In the second case above, if you did not get the SECOND RED we would have brb and continuing from here, the soonest we could stop is after two consecutive red draws, giving

\[ brbrr; \]

but now notice we could also terminate after exactly 5 draws if we got TWO blues in the at the start, followed by three consecutive red draws:

\[ bbrrr; \]

This shows ONE way to stop after 1 draw, ONE way to stop after 3 draws, and TWO WAYS to stop after 5 draws, so \(|B| = 4\).

Finally notice that you can also terminate after exactly SEVEN DRAWS as follows:

\[ bbbrrrr, bbrbrr, bbrbr, brbrbr, brrbr, brbbrr. \]