(Write up convincing answers to the questions marked by an asterisk (*), but not the unstarred questions. In fact some of the unstarred questions will be worked in the recitations. (**) is more challenging. If you are in the honors section, hand in solutions to (**) as well - otherwise you do not need to solve these questions (and they wont be graded). Your solutions will be due in class on Oct. 3, 2017. You can discuss solutions with others but you are expected to WRITE-UP THE SOLUTIONS ENTIRELY BY YOURSELF, without looking at any other students' solutions, nor receiving help with the writeup. When you finish your writeup, carefully describe WHO you collaborated with and in which problems, and then make the following PLEDGE: "This write-up is entirely my own work, with NO input from others."

1. (Sets) Decide whether the following statements are TRUE or FALSE and give a convincing argument to support your claim.

   (a) $(A \cup B) \setminus C = A \cup (B \setminus C)$.

   (b) (*) $(A \cup B) \setminus A = B$.

   (c) $(A \setminus B) \cup B = A$

   (d) (*) $(A \cup B) = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$.

   (e) $(A \cap B)^c \setminus A^c = B^c$

   (f) (*) $\bigcap_{i=1}^{n} A_i \subseteq A_1$.

   (g) $A_1 \subset \bigcup_{i=1}^{n} A_i$.

   (h) (*) $\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{m+n} A_i$, $m > 0$; i.e., $\bigcup_{i=1}^{k} A_i$ <u>increases</u> with k.

   (i) $\bigcap_{i=1}^{n} A_i \supset \bigcap_{i=1}^{m+n} A_i$, $m > 0$; i.e., $\bigcap_{i=1}^{k} A_i$ <u>strictly decreases</u> with k.

2. Let A,B and C be events in a sample space S. Express the following events - described in English - using the operators $\cup$, $\cap$, $\setminus$, and complement.

   (a) A occurs but neither B nor C do.

   (b) (*) At most two of the events occur.

   (c) (*) Exactly two of the events occur.

   (d) Exactly one of A, B, C occur, but not C.

   (e) (*) C occurs and at least another of them..

   (f) Not all of them occur.

3. (Random Experiments and Sample Spaces) For each of the following random experiments, carefully describe the sample space, S. Try to compute $|S|$, the size of S, and explain your answer.

   (a) (*) A coin is tossed 4 times.

   (b) A die is thrown 3 times.

   (c) (*) Five people enter the elevator in the basement of a building with 3 floors and the basement. Each states where he will get out (not the basement).

   (d) A box has 10 chips numbered 1 through 10. A chip is chosen and removed from the box, its value noted, and then it is returned to the box. Then one of the chips in the box is chosen, removed from the box, and its value noted.

   (e) (*) As in (d) but the box has six numbered chips and the chips are NOT returned to the box after they have been removed.

(f) (*) A die is tossed. If it shows an EVEN face, a coin is thrown. Otherwise (the die showed an ODD face), the die is thrown again and the results are written down.

(g) A 4 node, rooted binary tree is written down.

(h) (**) The hatcheck experiment with $n = 4$ people is performed (i.e., the hats of the 4 are randomly permuted, or redistributed, one hat to each person). Write down the sample space. How many of the outcomes are derangements (nobody gets their own hat)?

4. (Events) Carefully describe the events A, B, $A \cup B$, and $A \cap B$ in the following sample spaces from 3, above, and determine the sizes of these events.

(a) (*) In 3a, $A = \{$Head on the first and last tosses$\}$, $B = \{$at least 2 tails$\}$.

(b) In 3b, $A = \{$at least one even-sscore face$\}$, $B = \{$all faces 4 or more$\}$.

(c) In 3d, $A = \{$same chip both times$\}$, $B = \{$chip 10 is not chosen$\}$.

(d) (*) In 3f, $A = \{$the coin is NOT thrown$\}$, $B = \{$exactly one dice throw showed a four$\}$.

(e) (*) In 3g, $A = \{$the root has two children$\}$, $B = \{$the tree has height 4$\}$, the height being the number of edges on a longest path from the root to a leaf.

(f) (**) As in 3h, a box has 3 red and 5 black chips. In this experiment, you pick a chip, note its color, and return it to the box. The experiment continues *until* you have picked more red than black chips. (i) Describe the sample space and then let $A = \{$stop on third pick$\}$, $B = \{$stop before sixth pick$\}$.