1. (Sets) Decide whether the following statements are TRUE or FALSE and give a convincing argument to support your claim; if FALSE, try to find a counterexample.

(a) \((A \cup B) \setminus C = A \cup (B \setminus C)\).
(b) (*) \((A \cup B) \setminus A = B\).
(c) \((A \setminus B) \cup B = A\).
(d) (*) \((A \cup B) = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)\).
(e) \((A \cap B)^c \setminus A^c = B^c\).
(f) (*) \(\bigcap_{i=1}^{n} A_i \subseteq A_1\).
(g) \(A_1 \subset \bigcup_{i=1}^{n} A_i\).
(h) (*) \(\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{m+n} A_i, m > 0\); (i.e., \(\bigcup_{i=1}^{k} A_i\) increases with \(k\)).
(i) \(\bigcap_{i=1}^{n} A_i \subseteq \bigcap_{i=1}^{m+n} A_i, m > 0\); (i.e., \(\bigcap_{i=1}^{k} A_i\) increases with \(k\)).
(j) (**\) \(A \cap B = (A \cap B \cap C) \cup (A \cap B \cap D)\).

2. (Random Experiments and Sample Spaces) For each of the following random experiments, carefully describe the sample space, \(S\). Try to compute \(|S|\), the size of \(S\).

(a) (*) A coin is tossed 4 times.
(b) A die is thrown 3 times.
(c) (*) Three people enter the elevator in the basement of a building with 5 floors. Each states where he will get out.
(d) A box has 10 chips. The first is numbered with a “1”, the second with a “2”, etc. A chip is drawn from the box, its value noted, and then it is returned to the box. Then a second chip is drawn and its value noted.
(e) As in (d) but the chips are NOT returned to the box.
(f) (*) A die is tossed. If it shows an EVEN face, a coin is thrown. Otherwise (the die showed an ODD face), the die is thrown 2 more times and the results are written down.
(g) (*) A 4 node, rooted binary tree is written down.
(h) A convex hexagon is triangulated.
(i) The hatcheck experiment with \(n = 3\) hats. (In this experiment, \(n\) people check their hats. When someone comes to claim his/her hat, they are given one of the unclaimed hats, not necessarily their own. Thus, the hats are redistributed, one to each person.)
(j) (**\) The hatcheck experiment with \(n = 4\) hats. How many outcomes are derangements (nobody gets their own hat)? Any conjecture about the behaviour of the fraction of derangements in a random permutation of \(1, \ldots, n\)?

3. (Events) Carefully describe the events \(A, B, A \cup B,\) and \(A \cap B\) in the following sample spaces from 2, above, and determine the sizes of these events.

(a) (*) In 2a, \(A = \{\text{Head on the first and last tosses}\}\), \(B = \{\text{at least 2 tails}\}\).
(b) In 2b, $A = \{\text{at least one even-score face}\}$, $B = \{\text{all faces 4 or more}\}$.

(c) In 2d, $A = \{\text{same chip both times}\}$, $B = \{\text{chip 10 is not chosen}\}$.

(d) (*) In 2f, $A = \{\text{the coin is NOT thrown}\}$, $B = \{\text{exactly one dice throw showed a four}\}$.

(e) (*) In 2g, $A = \{\text{the root has two children}\}$, $B = \{\text{the tree has depth 4}\}$.

(f) (**) As in 2d, a box has 3 red and 5 black chips. In this experiment, you pick a chip, note its color, and return it to the box. The experiment continues until you have picked more red than black chips. (i) Describe the sample space and then let $A = \{\text{stop on third pick}\}$, $B = \{\text{stop before sixth pick}\}$. 