CS206 **HW1** **Sept. 5, 2009**

(Write up convincing answers to the questions marked by an asterisk (*). Some of the unstarred questions will be worked in the recitations. (**) indicates a more challenging problem. Your solutions are due in class on Wednesday, September 16, 2009.)

1. (Sets) Decide whether the following statements are TRUE or FALSE and give a convincing argument to support your claim; if FALSE, try to find a counterexample.
   (a) $(A \cup B) \setminus C = A \cup (B \setminus C)$.
   (b) (*) $(A \cup B) \setminus A = B$.
   (c) $(A \setminus B) \cup B = A$
   (d) (*) $(A \cup B) = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$.
   (e) $(A \cap B)^c \setminus A^c = B^c$
   (f) (*) $\bigcap_{i=1}^{n} A_i \subseteq A_1$.
   (g) $A_1 \subset \bigcup_{i=1}^{n} A_i$.
   (h) (*) $\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{m+n} A_i$, $m > 0$; (i.e., $\bigcup_{i=1}^{k} A_i$ increases with k).
   (i) $\bigcap_{i=1}^{n} A_i \subseteq \bigcap_{i=1}^{m+n} A_i$, $m > 0$; (i.e., $\bigcap_{i=1}^{k} A_i$ increases with k.
   (j) (**) $A \cap B = (A \cap B \cap C) \cup (A \cap B \cap D)$.

2. (Random Experiments and Sample Spaces) For each of the following random experiments, carefully describe the sample space, S. Try to compute $|S|$, the size of S.
   (a) (*) A coin is tossed 4 times.
   (b) A die is thrown 3 times.
   (c) (*) Three people enter the elevator in the basement of a building with 5 floors. Each states where he will get out.
   (d) A box has 10 chips. The first is numbered with a "1", the second with a "2", etc. A chip is drawn from the box, its value noted, and then it is returned to the box. Then a second chip is drawn and its value noted.
   (e) As in (d) but the chips are NOT returned to the box.
   (f) (*) A die is tossed. If it shows an EVEN face, a coin is thrown. Otherwise (the die showed an ODD face), the die is thrown 2 *more* times and the results are written down.
   (g) (*) A 4 node, rooted binary tree is written down.
   (h) A convex hexagon is triangulated.
   (i) The hatcheck experiment with $n = 3$ hats. (In this experiment, n people check their hats. When someone comes to claim his/her hat, they are given one of the unclaimed hats, not necessarily their own. Thus, the hats are redistributed, one to each person.)
   (j) (**) The hatcheck experiment with $n = 4$ hats. How many outcomes are derangements (nobody gets their own hat)? Any conjecture about the behaviour of the fraction of derangements in a random permutation of $1, \ldots, n$?

3. (Events) Carefully describe the events A, B, $A \cup B$, and $A \cap B$ in the following sample spaces from 2, above, and determine the sizes of these events.
   (a) (*) In 2a, $A = \{\text{Head on the first and last tosses}\}$, $B = \{\text{at least 2 tails}\}$.

(b) In 2b, $A = \{$at least one even-sscore face$\}$, $B = \{$all faces 4 or more$\}$.

(c) In 2d, $A = \{$same chip both times$\}$, $B = \{$chip 10 is not chosen$\}$.

(d) (*) In 2f, $A = \{$the coin is NOT thrown$\}$, $B = \{$exactly one dice throw showed a four$\}$.

(e) (*) In 2g, $A = \{$the root has two children$\}$, $B = \{$the tree has depth 4$\}$.

(f) (**) As in 2d, a box has 3 red and 5 black chips. In this experiment, you pick a chip, note its color, and return it to the box. The experiment continues *until* you have picked more red than black chips. (i) Describe the sample space and then let $A = \{$stop on third pick$\}$, $B = \{$stop before sixth pick$\}$.