This is hw3, the last one. It is due in class on April 30, 2018. You can talk with others about the problems, but I want to insist that you write up solutions *ENTIRELY* on your own. If you use a source like a book, a paper, or something taken from the web, carefully cite the sources, along with your solution. If you did discuss with others, please cite *THEM* in your solutions (maybe different people for different problems). Finally, when you hand in your solutions, write the pledge "I followed the rules for this assignment" and then sign your name. If you do not do this we will not grade your HW.

1. One formulation (standard?) for the Linear Programming (LP) problem in R^2 seeks a point $P^* = (x, y) \in R^2$ with minimum $y-$ coordinate that satisfies each of the following conditions:

   - $y \geq A_i x + B_i$, all $i \in I^+$
   - $y \leq A_i x + B_i$, all $i \in I^-$
   - and for each $i \in I^0$ either $-\infty \leq L_i \leq x$ or else $x \leq R_i \leq \infty$.

   Taking I^+, I^-, I^0 as disjoint subsets of the first n positive integers for which $|I^+|+|I^-|+|I^0| = n$, this version of LP seeks the "lowest point" (min y) in the intersection of n halfspaces in R^2 .

   N. Megiddo reformulated it taking $L = \max(L_i)$ and $R = \min(R_i)$ and stopping if $L > R$. Otherwise he defined

   $\mathcal{L}(x) = max_{i \in I^+} A_i x + B_i$, and

   $\mathcal{U}(x) = min_{i \in I^-} A_i x + B_i$,

   and rephrased the LP problem to seek $\min \mathcal{L}(x)$ subject to the conditions that $\mathcal{L}(x) \leq \mathcal{U}(x)$, and $x \in [L, R]$.

   (a) (5 pts) Argue that Megiddo's formulation is equivalent to the standard one because it will give the same answers as the standard one: (i) 'NO SOLUTION' when the n inequalities are inconsistent ("infeasible") or because there is no lowest point ("unbounded below"); (ii) "no unique solution" when the min y is on a horizontal line in I^+ with $A_i = 0$; or (iii) the unique lowest feasible point,

   (b) (10 pts) Use the description of the PRUNING steps (described briefly on the next page, for your convenience) to decide if its always true that **"IF $m = |I^+| + |I^-| (\leq n)$, THE ALGORITHM WILL PRUNE AT LEAST $m/4 + 1$ LINES", and EXPLAIN and JUSTIFY your answer**.

- “Pair” the lines in I^+ into at most $|I^+|/2$ distinct pairs (maybe one unmatched) and do the same with the lines in I^- (again, maybe one line unmatched). Write $m = |I^+| + |I^-| \leq n$ and there are $\leq m/2$ pairs.
- Maybe delete one line from some pairs: if the pair are parallel lines in I^+, delete the LOWER one, while if they are parallel and in I^-, delete the higher one (this is valid because the deleted lines CANNOT contain P^*, the lowest feasible point).
- For each remaining (complete) pair THAT MEETS OUTSIDE THE VERTICAL STRIP $[L, R]$ (i.e., meet at a point with $x < L$ or $x > R$):
  - if the pair is from $I+$ and meets at a point with $x < L$, delete the line with smaller slope - it cannot be the line in I^+ that determines $\mathcal{L}(\S)$ for $x \in [L, R]$; if the pair is from I^- and meets at a point with $x < L$, delete the line with bigger slope - it cannot be the line in I^- that determines $U(x)$ for $x \in [L, R]$.
  - if the pair is from I^+ and meets at a point with $x > R$, delete the line with bigger slope - it cannot be the line in I^+ that determines $\mathcal{L}(\S)$ for $x \in [L, R]$; if the pair is from I^- and meets at a point with $x > R$, delete the line with smaller slope - it cannot be the line in I^- that determines $U(x)$ for $x \in [L, R]$.
- compute $t^* \leftarrow$ median(x-coordinates of the intersection points of the remaining (complete, undeleted) pairs.
- Megiddo showed how to study the intersections of all non-vertical lines with the vertical $x = t^*$ and do one of the following: (i) conclude the problem is infeasible; (ii) give an optimal solution, a point on $x = t^*$; (iii) correctly decide which side of the line $x = t^*$ has a solution - if one exists for the LP. And now, for each of the at most $m/4$ pairs on the opposite side, one line may be deleted (it cannot be a line that determines P^*). This is the key PRUNING TOOL