**This is HW2, final edition. Currently it is due in class on April 11, 2018. A (*) denotes a harder question (*and is not required*), but you will get some extra credit if you give a decent answer. You can talk with others about the problems, but I want to insist that you write up solutions *entirely on your own. If you use a source like a book, a paper, or something taken from the web, carefully cite the sources, along with your solution. If you did discuss with others, please cite* them *in your solutions (maybe different people for different problems). Finally, when you hand in your solutions, write the pledge "I followed the rules for this assignment" and then sign your name. If you do not do this we will not grade your HW.***

1. $P_1, \ldots, P_n$ denotes the $n$ distinct vertices of a simple polygon - in order as we traverse the boundary of the polygon and with the interior always to the left. Write $P_i = (x_i, y_i)$.
   (a) Describe (no pseudo-code) a simple, efficient algorithm to decide if the polygon is x-monotone. What is its running time?
   (b) (*) Now repeat the above, replacing x-monotone by monotone in SOME direction.
2. Given points $P_1 = (-1, 1), P_2 = (2, -1)$ and $P_3 = (1, 2)$, we will use the duality that maps the point $P = (x, y)$ to the line $TP = \{(u, v) : v = xu + y\}$ and the line $\ell$ with equation $y = mx + b$ maps to the point $T\ell = (-m, b)$.
   (a) Carefully describe the dual of the triangle $\Delta P_1P_2P_3$, defined as the set of points on or above $\overrightarrow{P_1P_2}$, and on or left of $\overrightarrow{P_2P_3}$, and on or below $\overrightarrow{P_1P_3}$. In particular what is the dual of a point $Q$ exterior to the triangle? What is the dual of the point $Q = (P_1+P_2+P_3)/3$?
   (b) Carefully describe what is happening in the dual as you move around the boundary of $\Delta P_1P_2P_3$, going from $P_1$ to $P_2$ to $P_3$, then back to $P_1$, always moving along the edges of $\Delta P_1P_2P_3$.
3. Now consider the duality that maps a point $P = (x, y)$ to the line $TP = \ell(u, v)$ with equation $v = 2xu - y$; it also maps the line $L$ with equation $y = mx+b$ to the point $Q = TL = (m/2, -b)$. This dual is called the **polar dual" with respect to the parabola $y = x^2$.** To see what this means verify that:
   (a) A point $P = (x, x^2)$ that is ON the parabola $y = x^2$ dualizes to the line with equation $v = 2x * u - x^2$ that is tangent to the parabola $v = u^2$ at the point $P$. Then verify that
   (b) A point $P^{\pm} = (x, x^2 \pm b)$ that is ABOVE/BELOW the parabola $y = x^2$ by vertical distance $b > 0$ dualizes to the line $\ell(u, v)$ that is that is parallel to the tangent to the parabola $v = u^2$ at the point $P$ but BELOW/ABOVE that tangent by vertical distance $b > 0$; it has equation $v = 2xu - x^2 - b$.
   (c) Finally, repeat a) in the problem 2 - above - using this duality.

4. **Ham-Sandwich Problem - general case for $R^2$:** In the dual we are given $m = 2j + 1$ red lines $r_1, \dots r_m$ and $n = 2k + 1$ blue lines $b_1, \dots, b_n$, all $N = m + n$ lines in general position in $R^2$ (so all slopes distinct, no three lines incident in a point, no line vertical). The goal is to find a ham-sandwich cut, namely a RED/BLUE pair $r_s, b_t$ whose intersection $P = r_s \cap b_t$ is a point with $j$ red lines and $k$ blue lines above it, and the same numbers below).

   (a) By the Ham-sandwich Theorem there are an odd number of such points. Try to explain why this is true for the $m + n$ lines described above.

   (b) In class we discussed the special case in which the red lines all have negative slopes and the blue lines have positive slopes - this assures that there is a UNIQUE ham-sandwich cut for this case. Carefully explain why THIS is a true statement.

   (c) One of the tools used by the algorithm for the special, separated case was the "vertical query" which - given query-line $L$ with equation $x = t$ - decides if THE unique ham-sandwich vertex is to the LEFT, to the RIGHT, or ON $L$. We showed how to answer this query in $\Theta(m + n)$ time.

   Now you are asked to devise an algorithm that can answer the vertical query for the GENERAL CASE where the red and blue lines have arbitrary (*but distinct*) finite) slopes. Describe it carefully enough to be understandable (but NO pseudocode). It should say "ON" if there is a ham-sandwich vertex ON the query line $x = t$. Otherwise it should say "LEFT if it is certain that there is a ham-sandwich vertex to the left of the query line and if not, it should say RIGHT because it is certain that there is a ham-sandwich vertex to the right. Explain your algorithm and why it meets the above demands.

   State the complexity of your algorithm and explain your answer. What is the best lower bound for this task that you can give? Explain it?