

This is HW2, final edition. make the due-date a little later. Currently it is due in class on April 11, 2018. A (*) denotes a harder question (*and is not required*), but you will get some extra credit if you give a decent answer. You can talk with others about the problems, but I want to insist that you write up solutions *entirely on your own*. If you use a source like a book, a paper, or something taken from the web, carefully cite the sources, along with your solution. If you did discuss with others, please cite them in your solutions (maybe different people for different problems). Finally, when you hand in your solutions, write the pledge “I followed the rules for this assignment” and then sign your name. If you do not do this we will not grade your HW.

1. P_1, \dots, P_n denotes the n distinct vertices of a simple polygon - in order as we traverse the boundary of the polygon and with the interior always to the left. Write $P_i = (x_i, y_i)$.
 - (a) Describe (no pseudo-code) a simple, efficient algorithm to decide if the polygon is x-monotone. What is its running time?
 - (b) (*) Now repeat the above, replacing x-monotone by monotone in SOME direction.
2. Given points $P_1 = (-1, 1)$, $P_2 = (2, -1)$ and $P_3 = (1, 2)$, we will use the duality that maps the point $P = (x, y)$ to the line $TP = \{(u, v) : v = xu + y\}$ and the line ℓ with equation $y = mx + b$ maps to the point $T\ell = (-m, b)$.
 - (a) Carefully describe the dual of the triangle $\Delta P_1 P_2 P_3$, defined as the set of points on or above $\overrightarrow{P_1 P_2}$, and on or left of $\overrightarrow{P_2 P_3}$, and on or below $\overrightarrow{P_1 P_3}$. In particular what is the dual of a point Q exterior to the triangle? What is the dual of the point $Q = (P_1 + P_2 + P_3)/3$?
 - (b) Carefully describe what is happening in the dual as you move around the boundary of $\Delta P_1 P_2 P_3$, going from P_1 to P_2 to P_3 , then back to P_1 , always moving along the edges of $\Delta P_1 P_2 P_3$.
3. Now consider the duality that maps a point $P = (x, y)$ to the line $TP = \ell(u, v)$ with equation $v = 2xu - y$; it also maps the line L with equation $y = mx + b$ to the point $Q = TL = (m/2, -b)$. This dual is called the **polar dual** with respect to the parabola $y = x^2$. To see what this means verify that:
 - (a) A point $P = (x, x^2)$ that is ON the parabola $y = x^2$ dualizes to the line with equation $v = 2x * u - x^2$ that is tangent to the parabola $v = u^2$ at the point P . Then verify that
 - (b) A point $P^\pm = (x, x^2 \pm b)$ that is ABOVE/BELOW the parabola $y = x^2$ by vertical distance $b > 0$ dualizes to the line $\ell(u, v)$ that is that is parallel to the tangent to the parabola $v = u^2$ at the point P but BELOW/ABOVE that tangent by vertical distance $b > 0$; it has equation $v = 2xu - x^2 - b$.

(c) Finally, repeat a) in the problem 2 - above - using this duality.

4. **Ham-Sandwich Problem: general case for R^2 :** In the dual we are given $m = 2j + 1$ red lines r_1, \dots, r_m and $n = 2k + 1$ blue lines b_1, \dots, b_n , all $N = m + n$ lines in general position in R^2 (all slopes distinct, no three lines incident in a point, no line vertical). The goal is to find a ham-sandwich cut; i.e., a RED/BLUE pair r_s, b_t whose intersection $P = r_s \cap b_t$ is a point with j red lines and k blue lines above it, and the same numbers below). By the Ham-sandwich Theorem there are an odd number of such points.

(a) In class we discussed the special case in which the red lines all have negative slopes and the blue lines have positive slopes (so there is a UNIQUE ham-sandwich cut for this case). One of the tools used by that algorithm was the "vertical query" which - given line L with equation $x = t$, decides if THE ham-sandwich vertex is to the LEFT, to the RIGHT, or ON L , and we showed how to answer the query in $\Theta(m + n)$ time.

Now you are asked to devise an algorithm that can answer the vertical query for the general case when the red and blue lines have arbitrary (distinct) slopes, and to describe it carefully enough to be understandable (but NO pseudocode). What is the complexity of your algorithm? What is the best lower bound for this task that you can give, and explain it?