Problem. In Lecture 11, we mentioned the following result:

- **Palette Sparsification Theorem** [1]: Let $G = (V, E)$ be an $n$-vertex graph with maximum degree $\Delta$. Suppose for every vertex $v \in V$, we independently and uniformly at random sample $O(\log n)$ colors $L(v)$ from $\{1, \ldots, \Delta + 1\}$. Then, with high probability, there is a proper coloring of $G$ in which the color of every vertex $v$ is chosen from $L(v)$.

Based on this theorem, we showed how to design a semi-streaming algorithm for $(\Delta + 1)$ coloring.

Let us now consider sublinear time algorithms from the first half of the course. Use the palette sparsification theorem to give an $\tilde{O}(n^{3/2})$ query algorithm for $(\Delta + 1)$ coloring problem in the general query model (defined in Lecture 2). Note that for this problem, we only focus on the query complexity of the algorithms and not their time complexity although that can also be bounded (see [1]).

**Hint:** Give an $\tilde{O}(n^{3/2})$ query and time algorithm for finding the conflict graph defined in the context of the Palette Sparsification Theorem; then apply this theorem to finalize the proof.

References