Algorithms in the Field:
The Online Ads Example

S. Muthu Muthukrishnan
How to make Billions
How to make Billions

Doing Theory
How to make Billions

Doing Theory

& Applying it to Online Ads
Theory in Online Advertising

- Algorithm Design

[https://sites.google.com/site/algorithmsinthefield/]
Theory in Online Advertising

- Algorithm Design
- Engineering Algorithms
Theory in Online Advertising

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- Engineering Algorithms

Draw a straight line through 3 arbitrary points on the plane?
Theory in Online Advertising

- Algorithm Design

- Engineering Algorithms
  
  Draw a straight line through 3 arbitrary points on the plane?

- Algorithms in the Field.
  
  https://sites.google.com/site/algorithmsinthefield/
Online Ad Markets

- Ex: search, display, social, mobile, video, native, ...
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Social Ad Example
Online Ad Markets

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Native Ad Example: Click above
Online Ad Markets

- Ex: search, display, social, mobile, video, native, ...
- Online Ad Markets:
  - Where are the ads shown?
  - What are the creatives?
  - What is the targeting language?
  - What is a charging event?
  - What is under the hood: prediction, allocation, bidding, reporting and attribution, budget management, optimization and campaign goals.
Online Ad Allocation

- Advertisers $A_i$ set targeting and budget $B_i$ ahead of time.
- Each arriving impression $j$ has a set of $(i, j)$ of advertisers $A_i$ who fit.

$$\max \sum_{(i,j) \in E} y_{ij} \quad \text{s.t.} \quad \sum_i y_{ij} \leq 1 \quad \sum_j (i,j) \in E y_{ij} \leq 1 \quad y_{ij} = \{0, 1\}$$

$$\max \sum_{(i,j) \in E} b_{ij} y_{ij} \quad \text{s.t.} \quad \sum_i y_{ij} \leq 1 \quad \sum_j (i,j) \in E b_{ij} y_{ij} \leq B_i \quad y_{ij} = \{0, 1\}, \ b_{ij} \ll B_i$$

Table: Matching

Table: AdWords
AdWords

- Offline, NP Hard. $1 + \varepsilon$ approx. LP rounding.

- Online $1 - 1/e$ approx. worst case. The MSVV Algorithm:
  - For advertiser $i$ and for the arriving query $q$ define the scaled bid $\hat{b}_{iq} = b_{iq} \psi(f_i)$ where $f_i$ is the fraction of unspent budget of $i$, and $\psi(x) = 1 - e^{-x}$. For each arriving query we allocate it to the advertiser with the highest scaled bid.

- Online, iid or random order. $1 - \varepsilon$ approx when OPT is somewhat larger than any edge weight.
Ad Allocation: Display Ads

- Advertisers $A_i$ sets targeting and budget $B_i$ ahead of time.
- Each arriving impression $j$ has a set of $(i, j)$ of advertisers $A_i$ who fit.

$$\max \quad \sum_{(i,j) \in E} w_{ij} x_{ij}$$
$$\text{s.t.} \quad \sum_i x_{ij} \leq 1$$
$$\sum_j |(i,j) \in E| x_{ij} \leq N_i$$
$$x_{ij} = \{0, 1\}$$

No bounded competitive ratio possible ($N_i = 1, w_{1} = 100$, and $w_{2}$ is 100 or 1).
Ad Allocation: Display Ads

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- Each arriving impression $j$ has a set of $(i,j)$ of advertisers $A_i$ who fit.

\[
\begin{align*}
\text{max} & \quad \sum_{(i,j) \in E} w_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_i x_{ij} \leq 1 \\
& \quad \sum_{j|(i,j) \in E} x_{ij} \leq N_i \\
& \quad x_{ij} = \{0, 1\}
\end{align*}
\]

- No bounded competitive ratio possible ($N_1 = 1$, $w_1 = 100$ and $w_2$ is 10000 or 1).
Display Ads with Free Disposal

- Impressions satisfy **Free Disposal** property in Economics: optimize the weight of $N_i$ largest weight edges to $i$. 

Technical approach:

$x_{ij}$ denotes whether impression $j$ is one of the $N_i$ most valuable impressions assigned to $i$.

\[
\max_P (i; j) \sum_{E} w_{ij} x_{ij} \\
\text{s.t.} \quad \sum_{P} P_i x_{ij} \leq 1, \quad \sum_{P} P_j x_{ij} \leq 2 E x_{ij} \quad \forall j \leq N_i
\]

Algorithm: Maintain feasible solutions to primal and dual online. Assign $j$ to \( \text{argmax}_i f w_{ij} \). Update $i$ as exponential weighted average of $N_i$ largest weighted impressions.

Claim: \( (1 - \epsilon) = e \) approximation as $N_i! 1$. Used in DoubleClick. [FHKMS10]
Display Ads with Free Disposal

- Impressions satisfy Free Disposal property in Economics: optimize the weight of $N_i$ largest weight edges to $i$.
- Technical approach: $x_{ij}$ denotes whether impression $j$ is one of the $N_i$ most valuable impressions assigned to $i$.

$$\begin{align*}
\text{max} \quad & \sum_{(i,j) \in E} w_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_i x_{ij} \leq 1 \\
& \sum_{j \mid (i,j) \in E} x_{ij} \leq N_i
\end{align*}$$

$$\begin{align*}
\text{min} \quad & \sum_i N_i \beta_i + \sum_j \alpha_j \\
\text{s.t.} \quad & \beta_i + \alpha_j \geq w_{ij} \\
& x_{ij}, \beta_i, \alpha_j \geq 0
\end{align*}$$

Algorithm: Maintain feasible solutions to primal and dual online. Assign $j$ to $\arg\max_i w_{ij}$. Update $i$ as exponential weighted average of $N_i$ largest weighted impressions.

Claim: $(1+1)=e$ approximation as $N_i! \to 1$. Used in DoubleClick. [FHKMS10]
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\end{align*}$$

- Algorithm: Maintain feasible solutions to primal and dual online. Assign $j$ to $\arg\max_i \{w_{ij} - \beta_i\}$. Update $\beta_i$ as exponential weighted average of $N_i$ largest weighted impressions.

- Claim: $(1 - 1/e)$ approximation as $N_i \to \infty$. Used in DoubleClick. [FHKMS10]
Mobile apps advertise to get downloads.

- Advertisers have to pay per click, but
- Sales teams “sell” them some target (or threshold) cost per acquisition (tCPA).
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Put ad $i$ in for impression $j$ if $\frac{b_{ij} \cdot CTR_{ij}}{CTR_{ij} \cdot CVR_{i,j}} \leq tCPA_i$. Too conservative.
Mobile Ad Allocation

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  - Advertisers have to pay per click, but
  - Sales teams “sell” them some target (or threshold) cost per acquisition (tCPA).
- Put ad $i$ in for impression $j$ if $\frac{b_{ij} \cdot CTR_{ij}}{CTR_{ij} \cdot CVR_{i,j}} \leq tCPA_i$. Too conservative.

- Can set up the problem and look at Lagragians to derive a control parameter based method.
- Operational in Flurry mobile ad network. [Krushevskaja, M, Simpson, 15]
Ad Allocation with Intermediaries

- Display ad markets use ad exchanges which call brokers or ad networks which in turn represent adv.
- Mobile ad markets use multiple intermediate buy networks including ad exchanges.
- Challenge: intermediaries have capacity bottlenecks.
Prob that intermediary $I_i$ bids $k$ on impression $j$ is $p_{ijk}$.

$I_i$ has rate $\rho_i$ of impressions it can handle.

**Problem:** For each impression $j$, call out to set $S_j$ of networks and satisfy rate constraints. Maximize $\sum_j \text{max bid from } S_j$.

**Claim:** Assuming impressions are drawn from unknown distribution, there is an online algorithm with $1 - 1/e - \epsilon$ approximation. Primal dual approach, except solves a simple LP per impression. [Chakraborty, Even-dar, Guha, Mansour, M, 10]
Ad Allocation Summary

- Ad allocation: Matching and AdWords.
- Ad allocation with Free Disposal, for display ads.
- Ad allocation with secondary targets, for mobile app ads.
- Ad allocation with selective callout, for ad markets with intermediaries.
- Ad allocation with myriad variants, and other optimization problems in online ad markets.
Major Research Directions

- **Bulk Performance.** Allocate ads in such a way that advertisers see some “smooth” performance over time. Need a “dual” theory that controls supply. Eg., Yelp, Zillow.
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- **Micromarket Structure.** Advertisers cluster according to keywords, contexts, etc. Develop an economic theory for identifying these clusters.