1. The max-flow min-cut theorem states that in any graph, the size of a maximum \((s, t)\)-flow equals the size of the smallest \((s, t)\)-cut. If we pick an arbitrary vertex \(s\) and consider the partition induced by the global min-cut, then there has to exist at least one vertex \(t\) in the other part of the partition as \(s\). So we can run \((s, t)\)-max-flow procedures \(|V| - 1\) times (once for each choice of \(t\)) and pick the minimum of all the cuts obtained. By the previous argument, this cut has to be a global min-cut.

2. We will construct a weighted directed graph \(H\) from \(G\). The vertex set of \(H\) will contain the vertices of \(L\) and \(R\) along with two extra vertices \(s\) and \(t\). The original edges of \(G\) will appear in \(H\) as directed edges going from \(L\) to \(R\). We will also add the edges \(\{(s, l) | l \in L\}\) and \(\{(r, t) | r \in R\}\). Give all edges in \(H\) unit capacity. Now, the main point is that \(H\) has an \((s, t)\)-max-flow of size \(n\) if and only if \(G\) has a perfect matching.

To see this, suppose \(H\) has an \((s, t)\)-max-flow of size \(n\). Note that all the edges will have an integral flow through them (either 0 or 1) since each edge has an integral capacity and the algorithm would therefore never increment by a fractional amount. Moreover, all the edges from \(s\) to \(L\) are saturated and all the edges from \(R\) to \(t\) are saturated. No two saturated edges between \(L\) and \(R\) will ever share a vertex, since that will violate the conservation of flow at that vertex. So, a flow of size \(n\) has to reach from \(L\) to \(R\) through \(n\) independent edges, in other words, a perfect matching.

In the other direction, suppose \(G\) had a perfect matching. We can construct a flow of size \(n\) in \(H\) by saturating all the edges from \(s\) to \(L\), all the edges from \(R\) to \(t\), and all the edges corresponding to the perfect matching. It is easy to see that a flow of larger size will not be possible.