HW 2, Due Feb 14.

1. Assume you have access to a blackbox that tests if a given number is a prime. Design a fast randomized algorithm to pick a prime number in \([x, y]\), and analyze it in terms of \(x\) and \(y\).

2. Given strings \(S[1...n]\) and \(T[1..m]\), \(n \geq m\), find all locations \(i\) where \(s[i, ...i+m-1]\) and \(T\) differ in no more than \(k\) places, that is, for such \(i\), there are at most \(k\) positions \(j\) such that \(S[i + j − 1] \neq T[j]\). Use Karp-Rabin fingerprints.

3. Show an example of universal family of hash functions as defined in the class and prove that they satisfy the desired property.

4. Consider the process of tossing \(m\) balls into \(n\) bins. The tosses are uniformly at random and independent of each other, which implies that the probability that a ball falls into any given bin is \(1/n\).
   - What is the expected number of empty bins?
   - What is the probability of a particular bin having at least \(k\) balls? Provide a nice upper bound on this probability.