Sample Answers to Chapter 6

6.1
Given \( P(\text{cancer}) = .008 \), \( P(\neg\text{cancer}) = .992 \),
\( P(\text{+} | \text{cancer}) = .98 \), \( P(\text{-} | \text{cancer}) = .02 \),
\( P(\text{+} | \neg\text{cancer}) = .03 \), \( P(\text{-} | \neg\text{cancer}) = .97 \)

\[ P(\text{cancer} | \text{+}) \propto P(\text{+} | \text{cancer}) \cdot P(\text{cancer}) \]
\[ = P(\text{+} | \text{cancer}) \cdot P(\text{+} | \text{cancer}) \cdot P(\text{cancer}) \quad \text{(∵ the two tests are independent)} \]
\[ = (.98) \cdot (.98) \cdot (.008) \]
\[ = .0076832 \]

\[ P(\neg\text{cancer} | \text{+}) \propto P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer}) \]
\[ = P(\text{+} | \neg\text{cancer}) \cdot P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer}) \]
\[ = (.03) \cdot (.03) \cdot (.992) \]
\[ = .0008928 \]

Therefore,
\[ P(\text{cancer} | \text{+}) = \frac{.0076832}{.0076832 + .0008928} = .8959 \]
\[ P(\neg\text{cancer} | \text{+}) = \frac{.0008928}{.0076832 + .0008928} = .1041 \]

6.2
∵ \( P(\text{cancer} | \text{+}) + P(\neg\text{cancer} | \text{+}) = 1 \),
\[ P(\text{cancer} | \text{+}) = \frac{P(\text{+} | \text{cancer}) \cdot P(\text{cancer})}{P(\text{+})}, \]
\[ P(\neg\text{cancer} | \text{+}) = \frac{P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer})}{P(\text{+})}. \]

Therefore,
\[ P(\text{+} | \text{cancer}) \cdot P(\text{cancer}) / P(\text{+}) + P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer}) / P(\text{+}) = 1, \]
Implies,
\[ P(\text{+}) = P(\text{+} | \text{cancer}) \cdot P(\text{cancer}) + P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer}). \]
∴
\[ P(\text{cancer} | \text{+}) = \frac{P(\text{+} | \text{cancer}) \cdot P(\text{cancer})}{P(\text{+})} \]
\[ = (P(\text{+} | \text{cancer}) \cdot P(\text{cancer})) / (P(\text{+} | \text{cancer}) \cdot P(\text{cancer}) + P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer})), \]
and
\[ P(\neg\text{cancer} | \text{+}) = \frac{P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer})}{P(\text{+})} \]
\[ = P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer}) / (P(\text{+} | \text{cancer}) \cdot P(\text{cancer}) + P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer})). \]

The method to compute the posterior probability of cancer by normalizing the quantities \( P(\text{+} | \text{cancer}) \cdot P(\text{cancer}) \) and \( P(\text{+} | \neg\text{cancer}) \cdot P(\neg\text{cancer}) \) is valid.

6.6
Bayesian Belief Network:

![Bayesian Belief Network Diagram]

Conditional Probability Table associated with Wind:

<table>
<thead>
<tr>
<th></th>
<th>PlayTennis</th>
<th>¬PlayTennis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>1/3</td>
<td>0.6</td>
</tr>
<tr>
<td>Weak</td>
<td>2/3</td>
<td>0.4</td>
</tr>
</tbody>
</table>