Final practice

CS105: Great Insights in Computer Science
Michael L. Littman, Fall 2007

Rules

• Keep at least one empty seat between you and everyone else.

• Please write your answers directly on the pages.

• You may use your notes, your book, a calculator.

• You may not discuss the test with anyone else, including online sources (no cell phone, computer).

• You may ask clarification questions during the exam.

20 questions, 10 pts each
1. Exponentially Right?

Which of these sentences use the term “exponentially” correctly?

a. Once I got my cell phone, I was **exponentially** better at knowing what time it was.

b. The number of transistors on a chip has increased **exponentially** over the past twenty years.

c. Home prices in the northeast have been rising **exponentially**.

d. My new mac is **exponentially** more powerful than my previous one.

e. Now that I have kids, my life is **exponentially** more complex.

2. Random Questions

Consider the word “**valuable**”. If we repeatedly pick a random letter from the word...

a. How many times, on average, do we pick before we get the letter “b”?

b. How many times, on average, do we pick before getting one of the letters in the word “vat”?

c. How many times, on average, do we pick before choosing a vowel?
3. Huffman Coding

Find a Huffman code for “lolfootball”. How many bits does it take to write this string in your code?

4. Song Growth

How many syllables do you sing in $n$ verses of the following three songs? Choose from $O(n)$, $O(n \lg n)$, $O(n^2)$, and $O(n^2 \lg n)$.

Verses are numbered.

a.

b.

c.
4a. Animal Dept. Store

1. A kitty and a cat with mittens brand new got into an elevator (top floor). Then there were two: 1, 2!

2. Two little animals feeling carefree. A bull came from market. Then there were three: 1, 2, 3!

3. Three little animals waiting to arrive. In came a mule and a mare. And then there were five: 1, 2, 3, 4, 5!

4. Five little animals acting sedate. The Doggy family joined them. And then there were eight: 1, 2, 3, 4, 5, 6, 7, 8!

5. Eight little animals standing in a line. Shep the sheep sauntered in. And then there were nine: 1, 2, 3, 4, 5, 6, 7, 8, 9!

6. Nine little animals waiting yet again. Mr. Hog got on. And then there were ten: 1, 2, 3, 4, 5, 6, 7, 8, 9, one oh!

... (From Sesame Street. Assume for your analysis that between one and three animals get on per floor.)

4b. There’s A Hole

1. There’s a hole at the bottom of the sea. There’s a hole at the bottom of the sea. There’s a hole at the bottom of the sea.

2. There’s a log in the hole at the bottom of the sea. There’s a log in the hole at the bottom of the sea. There’s a log, there’s a log, there’s a log in the hole at the bottom of the sea.

3. There’s a bump on the log in the hole at the bottom of the sea. There’s a bump on the log in the hole at the bottom of the sea. There’s a bump, there’s a bump, there’s a bump on the log in the hole at the bottom of the sea.

4. There’s a frog on the bump on the log in the hole at the bottom of the sea. There’s a frog on the bump on the log in the hole at the bottom of the sea. There’s a frog, there’s a frog, there’s a frog on the bump on the log in the hole at the bottom of the sea.

5. There’s a leg on the frog on the bump on the log in the hole at the bottom of the sea. There’s a leg on the frog on the bump on the log in the hole at the bottom of the sea. There’s a leg, there’s a leg, there’s a leg on the frog on the bump on the log in the hole at the bottom of the sea.

...
4c. Ants Go Marching

The ants go marching one by one, hurrah, hurrah
The ants go marching one by one, hurrah, hurrah
The ants go marching one by one,
The little one stops to suck his thumb
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching two by two, hurrah, hurrah
The ants go marching two by two, hurrah, hurrah
The ants go marching two by two,
The little one stops to tie his shoe
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching three by three, hurrah, hurrah
The ants go marching three by three, hurrah, hurrah
The ants go marching three by three,
The little one stops to climb a tree
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching four by four, hurrah, hurrah
The ants go marching four by four, hurrah, hurrah
The ants go marching four by four,
The little one stops to shut the door
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching five by five, hurrah, hurrah
The ants go marching five by five, hurrah, hurrah
The ants go marching five by five,
The little one stops to take a dive
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching six by six, hurrah, hurrah
The ants go marching six by six, hurrah, hurrah
The ants go marching six by six,
The little one stops to pick up sticks
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching seven by seven, hurrah, hurrah
The ants go marching seven by seven, hurrah, hurrah
The ants go marching seven by seven,
The little one stops to pray to heaven
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

The ants go marching eight by eight, hurrah, hurrah
The ants go marching eight by eight, hurrah, hurrah
The ants go marching eight by eight,
The little one stops to shut the gate
And they all go marching down to the ground
To get out of the rain, BOOM! BOOM! BOOM!

5. Pathology

A. What’s the shortest path from G to F?
B. What’s a longest (cycle-free) path from G to C?
6. Mystery Routines

The scratch-like scripts below work on the 1 through \( n \) elements of list \( L \) and set the variable \( \text{answer} \) to zero (False) or one (True). Match each script to the question it answers.

A. Does the number -1 appear in the list?

B. Are all elements in the list ten or larger?

C. Is there an element in the list whose position matches the number (like the number 5 in the 5th position)?

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The Scripts

```
when I receive mystery1
repeat until \( n < 1 \)
  if \( L[n] < 10 \)
    set answer to 0
    stop script
  change n by -1
set answer to 1
stop script
```

```
when I receive mystery2
repeat until \( n < 1 \)
  if \( L[n] = -1 \)
    set answer to 1
    stop script
  change n by -1
set answer to 0
stop script
```

```
when I receive mystery3
repeat until \( n < 1 \)
  if \( L[n] = n \)
    set answer to 1
    stop script
  change n by -1
set answer to 0
stop script
```
What value will *total* have at the end of each script?

8. Algorithm Growth

You have a list of integers of length *n*. How fast can you answer the following questions if the list is unsorted? Sorted? Your choices are O(1), O(log *n*), O(*n*), O(*n*^2).

A. What is the second biggest number in the list?
   Unsorted: _______  Sorted: _______

B. Are there two numbers in the list that add up to 1000?
   Unsorted: _______  Sorted: _______

C. How many times does 74 appear in the list?
   Unsorted: _______  Sorted: _______

D. Is the product of the numbers divisible by 7?
   Unsorted: _______  Sorted: _______
9. Efficient List Algorithms


A. Is the product of these numbers divisible by 5?

B. Is the median of these 51 numbers bigger than 500?

10. Look It Up

Ask.com needs a way to create an index so they can quickly determine how many web pages contain any given word. They realize that they can solve their problem by sorting a list of all \( n \) words.

A. Using the solution of one problem (sorting) to solve another (indexing) is an example of what?

B. If they use selection sort to do their sorting, what is the running time?
11. Magic Word

A. What is the 6-letter magic word? (It’s actually two words, but just use the first.)

B. Use the tree on the next page to find a Huffman encoding for the word. If you don’t know the word, use “please”.

C. What does 10000100111010 spell?

Huffman Tree
12. 3-input Gate

Write a logical expression with A, B, C, and, or, not, as needed.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<td>True</td>
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</tbody>
</table>

D =

13. WIN @ NIM

In mini-nim, players take turns removing one or two objects from a pile. The goal is to be the one to remove the last object.

A. It’s your turn. There are 15 objects. How many should you take to win?

B. It’s your turn. There are 7 objects. How many should you take to win?
14. Subroutines

What will the sprite say when clicked?

15. Halt or Not

For what values of “?” does each of these scratch scripts halt?
16. Logical Question

If $a$ stands for “has green pants” and $b$ stands for “has open eyes”, give a logical expression that will split the group in half.

17. Graph Terms

A. List the nodes reachable from A.

B. List the source(s).
18. 10-exactly-one

A $k$-exactly-one gate outputs True if exactly one of its $k$ input bits are True. It can be made with one AND gate, one $k$-at-most-one gate, and one $k$-or gate.

A $k$-at-most-one gate outputs True if no more than one of its $k$ input bits are True. It can be made with $k(k-1)/2$ AND gates, one NOT gate, and one $k$-or gate.

A $k$-or gate outputs True if at least one of its $k$ input bits are True. It is made of $k-1$ OR gates.

How many AND gates are there in a 10-exactly-one gate?

19. Find The Bug

The number of described U.S. species of beetles is 23,700. If I’m thinking of one of them, how many guesses would it take for you to figure out which, using only yes/no questions?
Each program has a bug. Choose one description for each script.

A. Turns on the heat when it’s hot.
B. Turns on the heat at most once.
C. Leaves the heat always on or always off.
D. Turns off the heat at most once.
E. Never turns on the heat, no matter what.