# Representation and Reasoning Lecture Notes

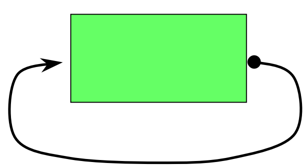
**Matthew Stone**

**CS 440, Fall 1999**

**Department of Computer Science and Center for Cognitive Science**

**Rutgers University**

## RECAP
## The Agent in its Environment



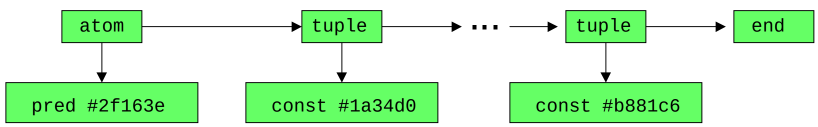
- **Agent has a representation**
  – a sentence in a formal language
- **corresponding to a real-world relationship**
  – via the semantics of the language
- **Agent uses the rep to make decisions**

## BASE CASE
## Representation

- **Representation is an atomic formula:**

$$\underbrace{p}_{\text{predicate}} \quad (\underbrace{t_1, \ \ldots \ \ t_n}_{\text{constants}}) \quad \text{(lower case str)}$$

  - constants represent objects in the world
  - predicate represents relation among them
  - written out for I/O with designer; really




## BASE CASE
## Semantics

- **Designer thinks up and specifies a model**
  - identifies objects and relations in the world needed to solve the problem
  - associates each constant with an object
  - associates each predicate with a relation
- **This determines**
  - what an atomic formula means
  - whether an agent's rep matches the world

BASE CASE

## Formal Semantics

- **To study representations, we formalize:**
  - Objects: universe or domain $D$
  - Consts: map $\phi$ from const to $D$
  - Preds: map $\pi$ from n-ary pred to $D^n \to \{T,F\}$

  Interpretation: $\langle D, \phi, \pi \rangle$
- **$p(t_1,\ldots,t_n)$ is true in interpretation iff**

  $\pi(p)\langle\phi(t_1),\ldots,\phi(t_n)\rangle = T$

VARIABLES AND RULES

## Representations

- **Terms are either constants or variables**
  - range over elements in the universe (strings beginning with caps or _)
- **Extended atoms:**

$$\underbrace{p}_{\text{predicate}} \; (\underbrace{t_1, \ldots \; t_n}_{\text{terms}})$$

  - expression without variables is ground

## VARIABLES AND RULES
## (Formal) Semantics

- **Variables interpreted by assignment**
  - temporarily links each variable to an object
  - formally, map $\rho$ from var to *D*

$$\delta(t_1,\rho) = \begin{cases} \phi(t_1) \text{ if } t_1 \text{ is a constant} \\ \rho(t_1) \text{ if } t_1 \text{ is a variable} \end{cases}$$

- **$p(t_1,\ldots,t_n)$ is true at $\rho$ iff**

$$\pi(p)\langle\delta(t_1,\rho),\ldots,\delta(t_n,\rho)\rangle = \top$$

## VARIABLES AND RULES
## Representations

- **Rules are a kind of compound formula**

$$\underbrace{h}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \ldots \wedge b_n}_{\text{body}}$$

  - head and body are atomic formulas
  - meaning: head is true whenever body is

## VARIABLES AND RULES
## (Formal) Semantics

- **$h \leftarrow b_1 \wedge \ldots \wedge b_n$ is true at $\rho$ iff**

  either h is true at $\rho$

  or some $b_i$ is not true at $\rho$
- **definite clause f is either rule or atom**
  - f is true iff

    for all $\rho$, f is true at $\rho$
- **knowledge base K: set of definite clauses**
  - K is true iff every $f \in K$ is true

## USING SEMANTICS
## Entailment

- **Formalization of information in KB**
- **$K \models f$ (read "K entails f") iff**
  - every interpretation where K is true is an interpretation where f is also true

## Proof

- **For us, a proof is a data structure**
  - that describes why a knowledge base entails some fact.
- **To describe the data structure**
  - A substitution $\sigma$ is a finite set of the form

$$\{ V_1 /t_1, \ldots V_n /t_n \}$$

  - The application of $\sigma$ to e, $e\sigma$, is e with occurrences of $V_i$ replaced by $t_i$

## Proof

- **A proof is a tree of judgments**
  - A judgment takes the form

$$K \rightarrow f$$

    read "f follows from K"
  - K is a knowledge base
  - f is a ground clause
- **Leaf is a judgment $K, e \rightarrow e\sigma$**

## Proof

- **Internal nodes**
  - we can combine proofs together thus:

$$\frac{\overset{P_0}{K \rightarrow h \leftarrow b_1 \wedge \ldots \wedge b_n} \qquad \overset{P_1}{K \rightarrow b_1} \quad \ldots \quad \overset{P_n}{K \rightarrow b_n}}{K \rightarrow h}$$

## Proof

- **When we have**

$$\overset{P}{K \rightarrow f}$$

**we say f is provable from K, or $K \vdash f$**

- **General result, for ground f:**

**$K \models f$ iff $K \vdash f$**