CS 205 Sections 07 and 08
Homework 5 – Accepted for grading 4/28

1. Create a finite automaton whose inputs are strings containing the letters *a*, *b* and *c*. You automaton should recongize any string that contains at least one *a* and at least one *b* but no *c*'s. Clearly label the states and transitions. Indicate the starting state and any final states.

2. An arcade game consists of three raised cylinders, labeled *A*, *B* and *C* respectively. The object of the game is to push down the cylinders in the proper sequence. A cylinder that is pushed down out of sequence will stay down, but the other two cylinders will pop up. When a cylinder is pushed down in its proper position in the sequence, all previous cylinders in the sequence will also stay down. The proper sequence is *BCA*. Design a finite automaton that models this arcade game.

   Hint. Use the states to represent which cylinders are down. There is only one final state.

3. Let A be a nonempty set such that $A^2 = A$.

   (a) Prove that $A^+ = A$.

   (b) Prove that $\lambda \in A$.

   Hint. Consider the cases $|A| = 1$ and $|A| > 1$. For the second case, consider a non-null string in A of minimal length.

   (c) Prove that $A^* = A$.

4. An infix expression is written in the form *exp op exp*, where *exp* is any expression and *op* is a binary operator. For this problem, assume that the expressions are either integers or one-letter variables. Also, assume that operators are one of the four standard arithmetic operators: $\{+,-,*,/\}$. Write a regular expression that matches input expressions with these restrictions.

5. Let L be a language over some vocabulary V. The *complement* of L is denoted by $\overline{L}$ and consists of all the strings in V^* that are not in L. Prove that if L is a regular language, then $\overline{L}$ is also a regular language.

   Hint. Use the fact that a language is a regular if and only if it is accepted by a finite state machine. Think about what the final and nonfinal states do in a finite state machine that accepts L.