1. Create a finite automaton whose inputs are strings containing the letters $a$, $b$ and $c$. You automaton should recognize any string that contains at least one $a$ and at least one $b$ but no $c$'s. Clearly label the states and transitions. Indicate the starting state and any final states.

2. An arcade game consists of three raised cylinders, labeled $A$, $B$ and $C$ respectively. The object of the game is to push down the cylinders in the proper sequence. A cylinder that is pushed down out of sequence will stay down, but the other two cylinders will pop up. When a cylinder is pushed down in its proper position in the sequence, all previous cylinders in the sequence will also stay down. The proper sequence is $BCA$. Design a finite automaton that models this arcade game.

*Hint.* Use the states to represent which cylinders are down. There is only one final state.

3. Let $A$ be a nonempty set such that $A^2 = A$.

   (a) Prove that $A^+ = A$.
   
   (b) Prove that $\lambda \in A$.

   *Hint.* Consider the cases $|A| = 1$ and $|A| > 1$. For the second case, consider a non-null string in $A$ of minimal length.

   (c) Prove that $A^* = A$.

4. An infix expression is written in the form $exp \ op \ exp$, where $exp$ is any expression and $op$ is a binary operator. For this problem, assume that the expressions are either integers or one-letter variables. Also, assume that operators are one of the four standard arithmetic operators: $\{+,-,\times,\div\}$. Write a regular expression that matches input expressions with these restrictions.

5. Let $L$ be a language over some vocabulary $V$. The complement of $L$ is denoted by $\bar{L}$ and consists of all the strings in $V^*$ that are not in $L$. Prove that if $L$ is a regular language, then $\bar{L}$ is also a regular language.

   *Hint.* Use the fact that a language is a regular if and only if it is accepted by a finite state machine. Think about what the final and nonfinal states do in a finite state machine that accepts $L$. 