CS 205 Sections 07 and 08

Homework 4 – Accepted for grading 4/12

1. Prove that whenever $p_1, \ldots p_n$ is a list of two or more propositions,

$$\neg(p_1 \vee p_2 \vee \ldots \vee p_n)$$

is logically equivalent to

$$\neg p_1 \wedge \neg p_2 \wedge \ldots \wedge \neg p_n$$

Use mathematical induction, and the fact that $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$ (De Morgan's law).

2. Prove by induction that if $a \equiv b(mod\ m)$ then $a^n \equiv b^n(mod\ m)$ for all $n \geq 0$.

3. Verify that the program segment

if $x < y$ **then**
    $m := x$
else
    $m := y$

is correct with respect to the initial asssertion **T** and the final assertion

$$(x \leq y \wedge m = x) \vee (x > y \wedge m = y)$$

4. This program computes quotients and remainders:

$r := a$
$q := 0$
while $r \geq d$
begin
    $r := r - d$
    $q := q + 1$
end

The program assumes that $d > 0$ and $a > 0$.

Prove that

$$d > 0 \ \wedge \ 0 \leq r \leq a \ \wedge \ a = dq + r$$

is a *loop invariant* for the **while** loop. In other words, show that if

$$d > 0 \ \wedge \ 0 \leq r \leq a \ \wedge \ a = dq + r \ \wedge \ r \geq d$$

is true at the beginning of any iteration of the loop, then

$$d > 0 \ \wedge \ 0 \leq r \leq a \ \wedge \ a = dq + r$$

is true afterwards.

5. Briefly, why does this invariant guarantee that the program can only terminate with a correct answer.