CS 205 Sections 07 and 08
Homework 3 – Accepted for grading 3/31

1. Each of the following items gives a condition on a function. Construct a function satisfying that condition. The domain and codomain of your function must be chosen from the sets

$$U = \{a,b,c\}, V = \{x,y,z\}, W = \{1,2\}$$

   (a) One-to-one but not onto.
   (b) Onto but not one-to-one.
   (c) One-to-one and onto.
   (d) Neither one-to-one nor onto.

2. Each of the following items specifies a function $f : \mathbb{N} \to \mathbb{N}$, and specifies certain of its properties. In each case, give a precise mathematical argument showing that the function satisfies the properties.

   (a) $f(x) = 2x$ — one-to-one but not onto.
   (b) $f(x) = \lfloor x/2 \rfloor$ — onto but not one-to-one.
   (c) $f(x) = \begin{cases} x-1 & \text{if } x \text{ is odd} \\ x+1 & \text{otherwise} \end{cases}$ — one-to-one and onto.

3. Let A, B and C be nonempty sets, and let $g : A \to B$ and $h : A \to C$ and let $f : A \to B \times C$ be defined by

$$f(x) = (g(x), h(x))$$

   Give a precise mathematical argument for each of the following statemetnts.

   (a) If f is onto, then g and h are onto.
   (b) It is not the case that f must be onto whenever g and h are onto.
   (c) If either g is one-to-one or h is one-to-one, then f is one-to-one.
   (d) It is possible for f to be one-to-one without either g or h being one-to-one.

4. Prove or disprove each of these statements about the floor and ceiling functions.

   (a) For all real numbers x,
$$\lfloor \lceil x \rceil \rfloor = \lfloor x \rfloor$$
   (b) $\lfloor x \rfloor = \lceil x \rceil$ if and only if x is an integer.
   (c) For all positive integers r,
$$\left\lfloor \log_2 \left\lfloor \frac{r+1}{2} \right\rfloor \right\rfloor = \left\lfloor \log_2 \left(\frac{r+1}{2} \right) \right\rfloor$$