1. Each of the following items gives a condition on a function. Construct a function satisfying that condition. The domain and codomain of your function must be chosen from the sets

\[ U = \{a, b, c\}, V = \{x, y, z\}, W = \{1, 2\} \]

(a) One-to-one but not onto.
(b) Onto but not one-to-one.
(c) One-to-one and onto.
(d) Neither one-to-one nor onto.

2. Each of the following items specifies a function \( f : \mathbb{N} \to \mathbb{N} \), and specifies certain of its properties. In each case, give a precise mathematical argument showing that the function satisfies the properties.

(a) \( f(x) = 2x \) — one-to-one but not onto.
(b) \( f(x) = \lfloor x/2 \rfloor \) — onto but not one-to-one.
(c) \( f(x) = \begin{cases} x - 1 & \text{if } x \text{ is odd} \\ x + 1 & \text{otherwise} \end{cases} \) — one-to-one and onto.

3. Let \( A, B \) and \( C \) be nonempty sets, and let \( g : A \to B \) and \( h : A \to C \) and let \( f : A \to B \times C \) be defined by

\[ f(x) = (g(x), h(x)) \]

Give a precise mathematical argument for each of the following statements.

(a) If \( f \) is onto, then \( g \) and \( h \) are onto.
(b) It is not the case that \( f \) must be onto whenever \( g \) and \( h \) are onto.
(c) If either \( g \) is one-to-one or \( h \) is one-to-one, then \( f \) is one-to-one.
(d) It is possible for \( f \) to be one-to-one without either \( g \) or \( h \) being one-to-one.

4. Prove or disprove each of these statements about the floor and ceiling functions.

(a) For all real numbers \( x \),

\[ \lfloor \lceil x \rceil \rfloor = \lfloor x \rfloor \]

(b) \( \lceil x \rceil = \lfloor x \rfloor \) if and only if \( x \) is an integer.

(c) For all positive integers \( r \),

\[ \left\lfloor \log_2 \left( \frac{r + 1}{2} \right) \right\rfloor = \left\lfloor \log_2 \left( \frac{r + 1}{2} \right) \right\rfloor \]