1. Consider the following assumptions about Superman.

   (a) If Superman were able and willing to prevent evil, he would do so.

   (b) If Superman were unable to prevent evil, he would be powerless.

   (c) If Superman were unwilling to prevent evil, he would be malevolent.

   (d) Superman does not prevent evil.

   (e) Superman is neither powerless nor malevolent.

   (Compare problem 77 of section 1.5 of Rosen, page 77.)

   These assumptions are contradictory. Give a *formal* proof of FALSE from these assumptions, using propositional logic. Use the following propositional symbols to represent your argument:

   | | |
   |---|---|
   | $A$ | Superman is able to prevent evil |
   | $W$ | Superman is willing to prevent evil |
   | $G$ | Superman prevents evil |
   | $P$ | Superman is powerless |
   | $M$ | Superman is malevolent |

   In addition, describe this proof in an *informal* English argument. Choose an order to introduce assumptions and the right level of detail so that the argument is convincing, easy to read, and clearly traces out the relationships between the assumptions of the argument, its intermediate conclusions, and the final result.

2. Consider the following argument:

> All radioactive substances either have a very short life or have medical value. No uranium isotope that is radioactive has a very short life. Therefore, if all uranium isotopes are radioactive, then all uranium isotopes have medical value.

This argument has two premises and one conclusion. Give a corresponding *formal* proof. Use the following predicate symbols to represent your argument:

$S(x)$   $x$ has a very short life
$M(x)$   $x$ has medical value
$U(x)$   $x$ is a uranium isotope
$R(x)$   $x$ is a radioactive substance

*Hint:* The overall structure of the argument involves universal generalization and conditional proof. Most of the work can be done by universal instantiation and modus ponens. The meaty step is to show that a generic uranium isotope does not have a very short life.

3. The following statement is a theorem of set theory:

$$\neg \exists S \forall x (x \in S \leftrightarrow x \notin x)$$

In English, this says no set contains exactly the sets that do not contain themselves. Give a *formal* proof of this statement. *Hint:* Use an *indirect proof*, and use existential instantiation; do case analysis based on whether the new set you consider does contain itself, or whether it doesn't.

In addition, describe this proof in an *informal* English argument. Choose how much mathematical notation and detail to retain so that your argument is convincing, easy to read, and clearly traces out the relationships between the assumptions of the argument, its intermediate conclusions, and the final result.