1. Formalize the following English sentences in propositional logic. Use the key provided.

(a) No shirt – no shoes – no service.
   \( I \): you wear a shirt
   \( O \): you wear shoes
   \( E \): you are served.

(b) The deluxe burger comes with fries and a coke.
   \( B \): you get a deluxe burger.
   \( F \): you get fries.
   \( C \): you get a coke.

(c) Delivery is available in New Brunswick for orders of $10 or more.
   \( N \): you order from within New Brunswick.
   \( T \): your order costs at least $10.
   \( D \): we will deliver your order.

(d) If you are not satisfied, you get your money back.
   \( S \): you are satisfied.
   \( M \): you get your money back.

(e) No refund without a receipt.
   \( M \): you get your money back.
   \( C \): you have a receipt.

2. Each item below offers a pair of compound propositions. In each case, say whether the two are logically equivalent. If they are not, give truth values for \( p, q, \) and \( r \) where the two compound propositions have different truth values.

(a) \( r \implies (\neg p \lor \neg q) \)
   \( \neg (p \land q \land \neg r) \)

(b) \( (p \lor q) \implies (\neg p \lor \neg q) \)
   \( p \implies \neg q \)

(c) \( p \implies (q \implies r) \)
   \( \neg r \implies \neg p \)

(d) \( (p \implies q) \implies (p \implies r) \)
   \( p \implies (q \implies r) \)

(e) \( \neg (p \implies q) \implies r \)
   \( (r \implies p) \implies q \)
3. Let the domain of discourse consist of all real numbers. Let $P(x, y)$ mean $yx^2 = y^3$. Which of the following propositions are true, and which are false?

(a) $P(0, 0)$
(b) $P(-1, -1) \rightarrow P(0, 1)$
(c) $P(1, 2) \rightarrow P(1, -1)$
(d) $\forall x P(x, x)$
(e) $\forall x P(x, -x)$
(f) $\exists x P(x, 2x)$
(g) $\exists x \forall y P(x, 2x)$
(h) $\exists y \forall x P(x, y)$
(i) $\exists y \forall x P(x, y)$
(j) $\forall x \forall y \forall z (P(x, y) \rightarrow P(xz, yz))$

4. Formalize the following English sentences in predicate logic. Use the key provided. Use the constant $a$ to represent the store about which these rules are true.

(a) We honor competitors’ coupons.
   $M(x, y): x$ competes with $y$.
   $C(x, y): x$ is a coupon for store $y$.
   $H(x, y): x$ honors $y$.

(b) None of our pizzas contain any artificial ingredients.
   $Z(x): x$ is a pizza.
   $S(x, y): x$ sells $y$.
   $A(x): x$ is artificial.
   $C(x, y): x$ contains $y$.

(c) Buy one pizza get one free.
   $P(x, y, z): x$ pays $yz$ dollars.
   $G(x, y, o): x$ gives $y$ object $o$.
   $Z(x): x$ is a pizza.
   $F(z): z$ is the full price for a pizza.

(d) Opened CDs can only be exchanged for another copy of the same title.
   $C(x): x$ is a CD.
   $O(x): x$ has been opened.
   $T(x, t):$ the title of $x$ is $t$ (the type of recording).
   $E(x, y, o, p): x$ gives $y$ object $o$ and $y$ gives $x$ object $p$ in exchange.

(e) Our prices are the lowest.
   $P(o, x, z):$ the price of product $o$ in store $x$ is $z$ dollars.