1. Formalize the following English sentences in propositional logic. Use the key provided.

(a) No shirt – no shoes – no service.
   \( I \): you wear a shirt
   \( O \): you wear shoes
   \( E \): you are served.
   Answer:
   \( \neg I \lor \neg O \rightarrow \neg E \)

(b) The deluxe burger comes with fries and a coke.
   \( B \): you get a deluxe burger.
   \( F \): you get fries.
   \( C \): you get a coke.
   Answer:
   \( B \rightarrow F \land C \)

(c) Delivery is available in New Brunswick for orders of $10 or more.
   \( N \): you order from within New Brunswick.
   \( T \): your order costs at least $10.
   \( D \): we will deliver your order.
   Answer:
   \( N \land T \rightarrow D \)
   Also OK:
   \( D \rightarrow N \land T \)

(d) If you are not satisfied, you get your money back.
   \( S \): you are satisfied.
   \( M \): you get your money back.
   Answer:
   \( \neg S \rightarrow M \)

(e) No refund without a receipt.
   \( M \): you get your money back.
   \( C \): you have a receipt.
   Answer:
   \( \neg C \rightarrow \neg M \)
2. Each item below offers a pair of compound propositions. In each case, say whether the two are logically equivalent. If they are not, give truth values for \( p \), \( q \), and \( r \) where the two compound propositions have different truth values.

(a) \( r \rightarrow (\neg p \lor \neg q) \)
\( \neg(p \land q \land \neg r) \)
Answer: Not equivalent.
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(b) \( (p \lor q) \rightarrow (\neg p \lor \neg q) \)
\( p \rightarrow \neg q \)
Answer: Equivalent.
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(c) \( p \rightarrow (q \rightarrow r) \)
\( \neg r \rightarrow \neg p \)
Answer: Not equivalent.
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(d) \((p \rightarrow q) \rightarrow (p \rightarrow r)\)
\(p \rightarrow (q \rightarrow r)\)
Answer: Equivalent.

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(e) \(\neg (p \rightarrow q) \rightarrow r\)
\((r \rightarrow p) \rightarrow q\)
Answer: Not equivalent.

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3. Let the domain of discourse consist of all real numbers. Let \( P(x, y) \) mean \( yx^2 = y^3 \). Which of the following propositions are true, and which are false?

(a) \( P(0,0) \)
   Answer: true.

(b) \( P(-1,-1) \rightarrow P(0,1) \)
   Answer: false.

(c) \( P(1,2) \rightarrow P(1,-1) \)
   Answer: true.

(d) \( \forall x P(x,x) \)
   Answer: true.

(e) \( \forall x P(x,-x) \)
   Answer: true.

(f) \( \exists x P(x,2x) \)
   Answer: true.

(g) \( \exists x \lnot P(x,2x) \)
   Answer: true.

(h) \( \exists x \forall y P(x,y) \)
   Answer: false.

(i) \( \exists y \forall x P(x,y) \)
   Answer: true.

(j) \( \forall x \forall y \forall z (P(x,y) \rightarrow P(xz,yz)) \)
   Answer: true.
4. Formalize the following English sentences in predicate logic. Use the key provided. Use the constant \( a \) to represent the store about which these rules are true.

(a) We honor competitors’ coupons.
\[
M(x, y) \colon x \text{ competes with } y.
\]
\[
C(x, y) \colon x \text{ is a coupon for store } y.
\]
\[
H(x, y) \colon x \text{ honors } y.
\]

Answer:
\[
\forall s \forall c(M(s, a) \land C(c, s) \to H(a, c))
\]

(b) None of our pizzas contain any artificial ingredients.
\[
Z(x) \colon x \text{ is a pizza.}
\]
\[
S(x, y) \colon x \text{ sells } y.
\]
\[
A(x) \colon x \text{ is artificial.}
\]
\[
C(x, y) \colon x \text{ contains } y.
\]

Answer:
\[
\neg \exists p \exists i(Z(p) \land S(a, p) \land C(p, i) \land A(i))
\]

(c) Buy one pizza get one free.
\[
P(x, y, z) \colon x \text{ pays } y \text{ } z \text{ dollars.}
\]
\[
G(x, y, o) \colon x \text{ gives } y \text{ object } o.
\]
\[
Z(x) \colon x \text{ is a pizza.}
\]
\[
F(z) \colon z \text{ is the full price for a pizza.}
\]

Answer:
\[
\forall x \forall z(P(x, a, z) \land F(z) \to \exists p \exists q(Z(p) \land Z(q) \land p \neq q \land G(a, x, p) \land G(a, x, q)))
\]

(d) Opened CDs can only be exchanged for another copy of the same title.
\[
C(x) \colon x \text{ is a CD.}
\]
\[
O(x) \colon x \text{ has been opened.}
\]
\[
T(x, t) \colon \text{the title of } x \text{ is } t \text{ (the type of recording).}
\]
\[
E(x, y, o, p) \colon x \text{ gives } y \text{ object } o \text{ and } y \text{ gives } x \text{ object } p \text{ in exchange.}
\]

Answer:
\[
\forall x \forall o \forall p \forall t(E(x, a, o, p) \land C(o) \land O(o) \land T(o, t) \to C(p) \land T(p, t))
\]

(e) Our prices are the lowest.
\[
P(o, x, z) \colon \text{the price of product } o \text{ in store } x \text{ is } z \text{ dollars.}
\]

Answer:
\[
\forall o \forall x \forall y \forall z(P(o, x, z) \land P(o, a, y) \to y \leq z)
\]