CS 205 Sections 07 and 08
Homework 1 – Accepted for grading 2/18
Answer Key

1. Formalize the following English sentences in propositional logic. Use the key provided.

   (a) No shirt – no shoes – no service.
   I: you wear a shirt
   O: you wear shoes
   E: you are served.

   Answer:
   $\neg I \vee \neg O \rightarrow \neg E$

   (b) The deluxe burger comes with fries and a coke.
   B: you get a deluxe burger.
   F: you get fries.
   C: you get a coke.

   Answer:
   $B \rightarrow F \wedge C$

   (c) Delivery is available in New Brunswick for orders of $10 or more.
   N: you order from within New Brunswick.
   T: your order costs at least $10.
   D: we will deliver your order.

   Answer:
   $N \wedge T \rightarrow D$

   Also OK:
   $D \rightarrow N \wedge T$

   (d) If you are not satisfied, you get your money back.
   S: you are satisfied.
   M: you get your money back.

   Answer:
   $\neg S \rightarrow M$

   (e) No refund without a receipt.
   M: you get your money back.
   C: you have a receipt.

   Answer:
   $\neg C \rightarrow \neg M$

2. Each item below offers a pair of compound propositions. In each case, say whether the two are logically equivalent. If they are not, give truth values for p, q, and r where the two compound propositions have different truth values.

   (a) $r \rightarrow (\neg p \vee \neg q)$
   $\neg(p \wedge q \wedge \neg r)$

   Answer: Not equivalent.

   Truth table:

p	q	r	$r \rightarrow (\neg p \vee \neg q)$	$\neg(p \wedge q \wedge \neg r)$	
t	t	t	f	t	*
t	t	f	t	f	*
t	f	t	t	t	
t	f	f	t	t	
f	t	t	t	t	
f	t	f	t	t	
f	f	t	t	t	
f	f	f	t	t	

   (b) $(p \vee q) \rightarrow (\neg p \vee \neg q)$
   $p \rightarrow \neg q$

   Answer: Equivalent.

   Truth table:

p	q	$(p \vee q) \rightarrow (\neg p \vee \neg q)$	$p \rightarrow \neg q$
t	t	f	f
t	f	t	t
f	t	t	t
f	f	t	t

   (c) $p \rightarrow (q \rightarrow r)$
   $\neg r \rightarrow \neg p$

   Answer: Not equivalent.

   Truth table:

p	q	r	$p \rightarrow (q \rightarrow r)$	$\neg r \rightarrow \neg p$	
t	t	t	t	t	
t	t	f	f	f	
t	f	t	t	t	
t	f	f	t	f	*
f	t	t	t	t	
f	t	f	t	t	
f	f	t	t	t	
f	f	f	t	t	

(d) $(p \to q) \to (p \to r)$
$p \to (q \to r)$

Answer: Equivalent.

Truth table:

p	q	r	$p \to (q \to r)$	$\neg r \to \neg p$
t	t	t	t	t
t	t	f	f	f
t	f	t	t	t
t	f	f	t	t
f	t	t	t	t
f	t	f	t	t
f	f	t	t	t
f	f	f	t	t

(e) $\neg(p \to q) \to r$
$(r \to p) \to q$

Answer: Not equivalent.

Truth table:

p	q	r	$\neg(p \to q) \to r$	$(r \to p) \to q$	
t	t	t	t	t	
t	t	f	t	f	
t	f	t	t	f	*
t	f	f	f	f	
f	t	t	t	t	
f	t	f	t	t	
f	f	t	t	t	
f	f	f	t	f	*

3. Let the domain of discourse consist of all real numbers. Let $P(x,y)$ mean $yx^2 = y^3$. Which of the following propositions are true, and which are false?

   (a) $P(0,0)$
   Answer: true.

   (b) $P(-1,-1) \rightarrow P(0,1)$
   Answer: false.

   (c) $P(1,2) \rightarrow P(1,-1)$
   Answer: true.

   (d) $\forall x P(x,x)$
   Answer: true.

   (e) $\forall x P(x,-x)$
   Answer: true.

   (f) $\exists x P(x,2x)$
   Answer: true.

   (g) $\exists x \neg P(x,2x)$
   Answer: true.

   (h) $\exists x \forall y P(x,y)$
   Answer: false.

   (i) $\exists y \forall x P(x,y)$
   Answer: true.

   (j) $\forall x \forall y \forall z (P(x,y) \rightarrow P(xz,yz))$
   Answer: true.

4. Formalize the following English sentences in predicate logic. Use the key provided. Use the constant a to represent the store about which these rules are true.

   (a) We honor competitors' coupons.
   $M(x,y)$: x competes with y.
   $C(x,y)$: x is a coupon for store y.
   $H(x,y)$: x honors y.

   Answer:
   $\forall s \forall c(M(s,a) \wedge C(c,s) \rightarrow H(a,c))$

   (b) None of our pizzas contain any artificial ingredients.
   $Z(x)$: x is a pizza.
   $S(x,y)$: x sells y.
   $A(x)$: x is artificial.
   $C(x,y)$: x contains y.

   Answer:
   $\neg\exists p \exists i(Z(p) \wedge S(a,p) \wedge C(p,i) \wedge A(i))$

   (c) Buy one pizza get one free.
   $P(x,y,z)$: x pays y z dollars.
   $G(x,y,o)$: x gives y object o.
   $Z(x)$: x is a pizza.
   $F(z)$: z is the full price for a pizza.

   Answer:
   $\forall x \forall z(P(x,a,z) \wedge F(z) \rightarrow \exists p \exists q(Z(p) \wedge Z(q) \wedge p \neq q \wedge G(a,x,p) \wedge G(a,x,q)))$

   (d) Opened CDs can only be exchanged for another copy of the same title.
   $C(x)$: x is a CD.
   $O(x)$: x has been opened.
   $T(x,t)$: the title of x is t (the type of recording).
   $E(x,y,o,p)$: x gives y object o and y gives x object p in exchange.

   Answer:
   $\forall x \forall o \forall p \forall t(E(x,a,o,p) \wedge C(o) \wedge O(o) \wedge T(o,t) \rightarrow C(p) \wedge T(p,t))$

   (e) Our prices are the lowest.
   $P(o,x,z)$: the price of product o in store x is z dollars.

   Answer:
   $\forall o \forall x \forall y \forall z(P(o,x,z) \wedge P(o,a,y) \rightarrow y \leq z)$