CS 520: Introduction to Artificial Intelligence

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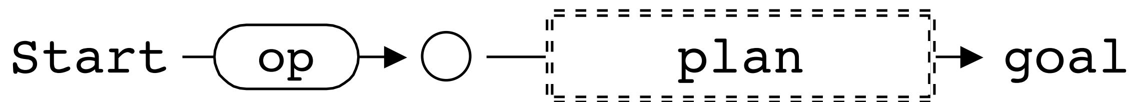
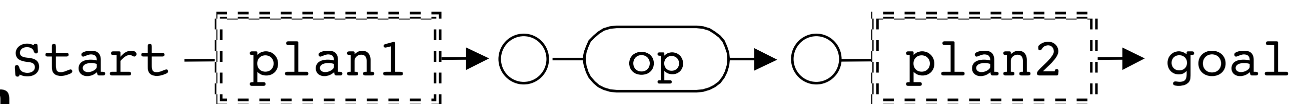
Lecture 16: Uncertainty

Review: Building Plans

- **Progression**
  - **Choose first step, what state does that result in?**
    - **Plan from that state to goal**



- **Means-Ends**
  - **Choose an operator that reduces start-goal difference**
  - **Plan from start to preconditions of operator**
  - **Plan from result of operator to goal**



- **Regression**
  - **Choose last step, what conditions needs to be true before it to**
  - **ensure goal met after it?**
  - **Plan from start to those conditions**

Sources of Uncertainty

- **Incomplete knowledge of the world state**
  - **Which room is the wumpus in?**
- **Uncertainty in operator effects**
  - **Command: turn 90 degrees right**
  - **Result: robot turns 89.35 degrees right**
- **Theoretical vs practical uncertainty**
  - **We have no way to know, v.s.**
  - **It would cost too much to know**
    - **Human time**
    - **Computer time**

Modeling Uncertainty

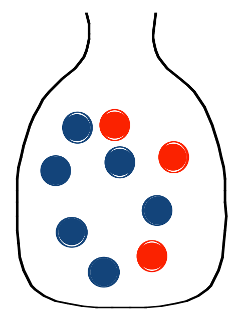
- **Degree of belief**
  - **Historical interest: approximations of probability**
- **Fuzzy logic**
  - **Oriented towards linguistic imprecision**
    - **It is hot vs it is warm vs it is sweltering**
- **Probability**

Acting under uncertainty

- **Probability: methods for representing and reasoning about uncertainty of knowledge**
- **Utility theory: a way to represent and reason about the** *desirability* **of different outcomes**
- **Decision theory: A way to combine probability and utility to decide what to do.**

Probability

- **Bag with 10 marbles: 3 red, 7 blue**



  - **Reach in, take one, put it back**
  - **Repeat lots of times.**
  - **What fraction red? About .3**
  - **P(red) = .3**

Probability Distribution

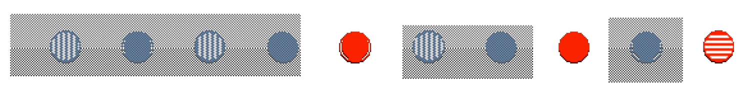
- **The probability for each value of a random variable**

    if color = (red, blue)

    P(color) = (.3, .7)

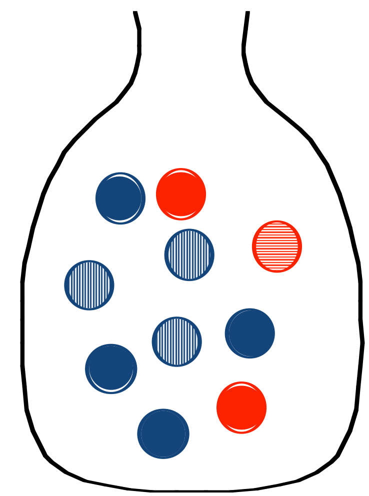
Conditional Probability

- Marbles are red or blue, striped or solid
- Take marble, look at it, put it back
- Ignore non-red
- What fraction are striped?



P(striped | red) = 1/3

- P(red | striped) = 1/4



Conditional Probability

- P(A | B) = P(A $\wedge$ B) / P(B)

  .1 P(red $\wedge$ striped)

  .4 P(striped)

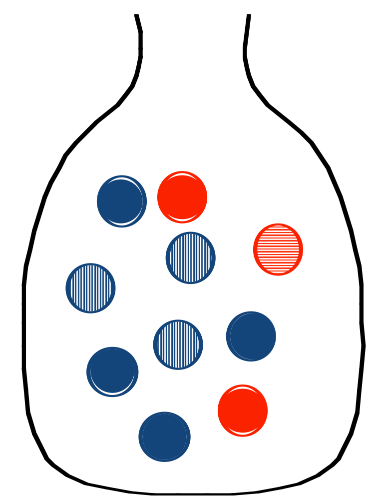
  .1/.4 = 1/4 = P(red | striped)

  .1 P(red $\wedge$ striped)

  .3 P(red)

  .1/.3 = 1/3 = P(striped | red)

- Equivalently: P(A^B) = P(A | B) * P(B)



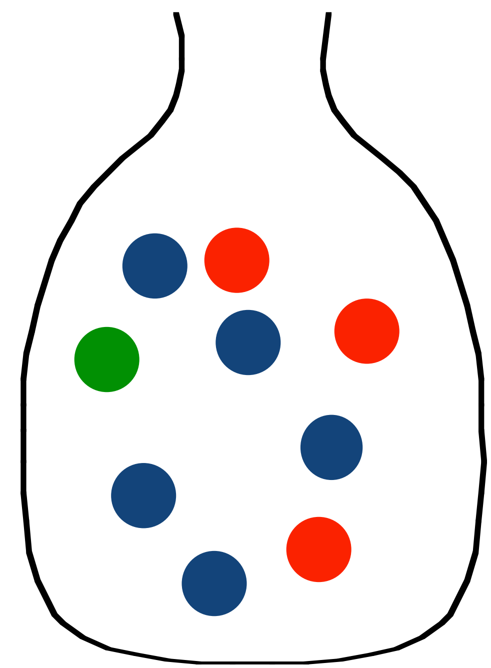
Basic Properties

- $0 \leq P(A) \leq 1$
- P(true) = 1

  P(red blue green) = 1

- P(false) = 0

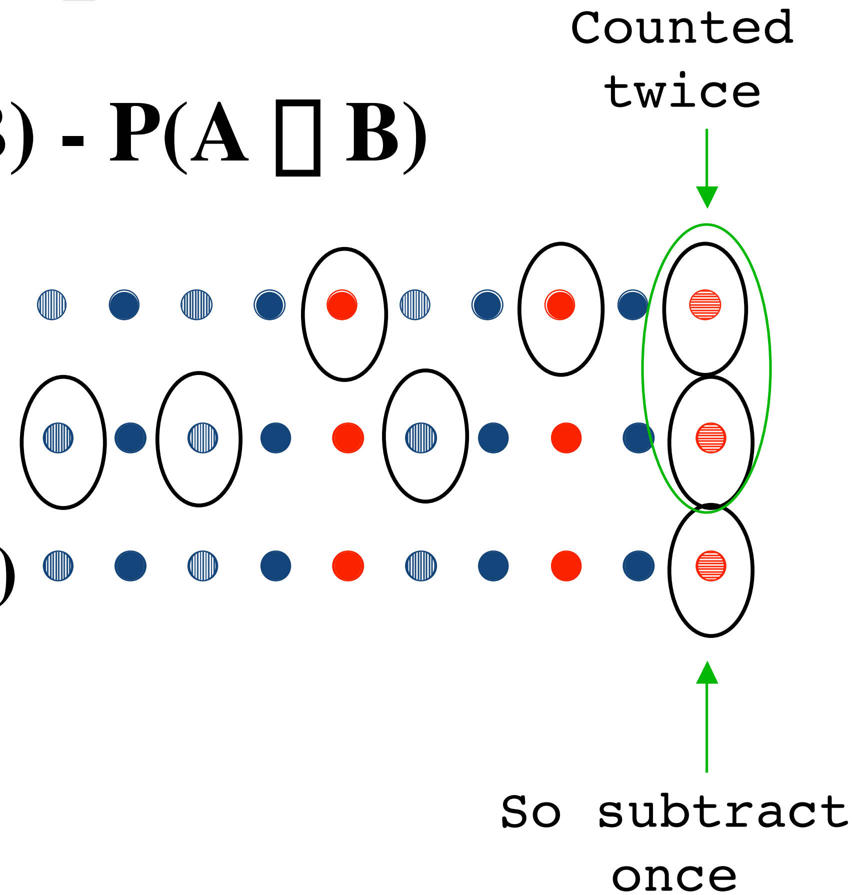
  P(black) = 0



Basic Properties

- P(A B) = P(A) + P(B) - P(A □ B)

.3	P(red)
+ .4	P(striped)
- .1	P(red □ striped)



P(red striped) = .6

Example Proofs

- **Prove $P(true \mid A) = 1$**
- **Proof:**

  $P(true \mid A) = P(true \wedge A)/P(A) = P(A)/P(A) = 1$

- **Prove: $P(B|A)+P(\neg B|A)= 1$**
- **Proof:**

  $P(B|A)+P(\neg B|A) = P(B \vee \neg B |A) + P(B \wedge \neg B|A)$

  $= P(true \mid A) + P(false \mid A) = 1 + 0$

Example Proof

- **Prove:**

  $P(A) = P(A \mid B) * P(B) + P(A \mid \neg B) * P(\neg B)$

- **Proof:**

  $P(B \mid A) + P(\neg B \mid A) = P(B \vee \neg B \mid A) = P(true \mid A) = 1$

  $P(B \mid A) + P(\neg B \mid A) =$

  $P(A \mid B) * P(B) / P(A) + P(A \mid \neg B) * P(\neg B) / P(A) =$

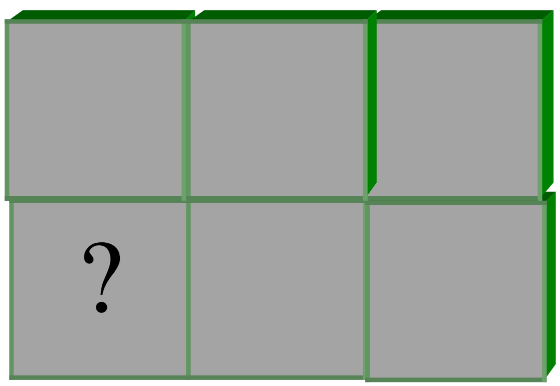
  $(P(A \mid B) * P(B) + P(A \mid \neg B) * P(\neg B)) / P(A)$

Example Use: Mine Sweeper

- Grid of cells, some have mines
- Probe a cell:
  - If has a mine, lose
  - Else tells number of neighboring mines
- Variables:
  - $M(x, y)$: boolean: mine
  - $C(x, y)$: neighbor mine count
  - N: total mines

Mine Sweeper

N = 3



P(M(1, 1)) =

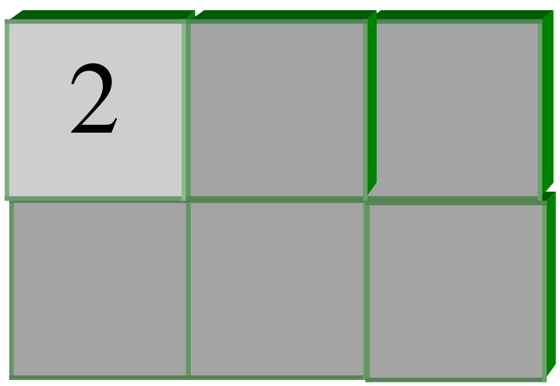
$$\frac{\text{\# ways of choosing 2 of 5}}{\text{\# ways of choosing 3 of 6}}$$

$$= \frac{5!/(2!3!)}{6!/(3!3!)}$$

$$= 10/20 = .5$$

Mine Sweeper

N = 3



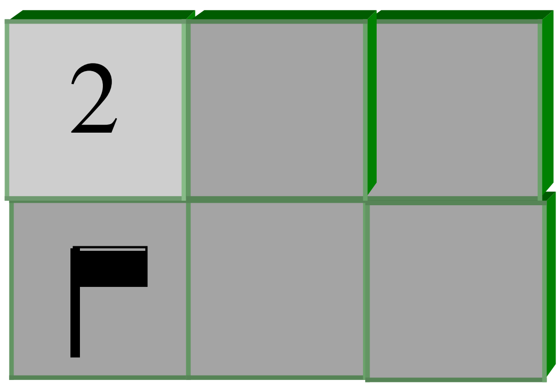
P(C(1,2)=2 ^ ~M(1,2)) =

$$\frac{\text{choose}(2, 3) * \text{choose}(1,2)}{\text{choose}(3, 6)}$$

= 3*2/20 = .3

Mine Sweeper

N = 3



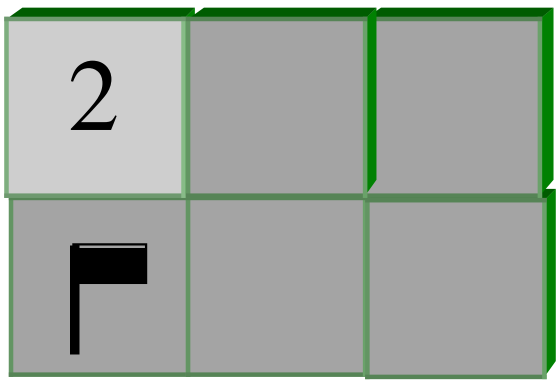
P(C(1,2)=2 ^ ~M(1,2) ^ M(1,1))=

$$\frac{\text{choose}(1, 2) * \text{choose}(1,2)}{\text{choose}(3, 6)}$$

= 2 * 2 / 20 = .2

Mine Sweeper

N = 3

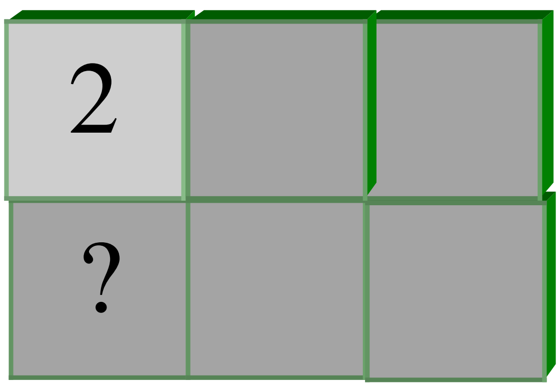


$$P(\, C(1,2)=2 \wedge \sim M(1,2) \mid M(1,1) \,) = \frac{P(\, C(1,2)=2 \wedge \sim M(1,2) \wedge M(1,1) \,)}{P(\, M(1,1) \,)}$$

$$= .2/.5 = 2/5$$

Mine Sweeper

N = 3



P(M(1,1) | C(1,2)=2 ^ ~M(1,2))=

P(C(1,2)=2 ^ ~M(1,2) | M(1,1))

* P(M(1,1))

/ P(C(1,2)=2 ^ ~M(1,2))

= 2/5 * 1/2 / 3/10

= 2/3

Mine Sweeper

- **Known:**

  P(M(1, 1) v M(2, 1)) =1

  P(M(1, 1) ^ M(2, 1)) = 0

  P(M(1, 1)) = P(M(2, 1))

- **Conclude:**

  1 = P(M(1, 1))

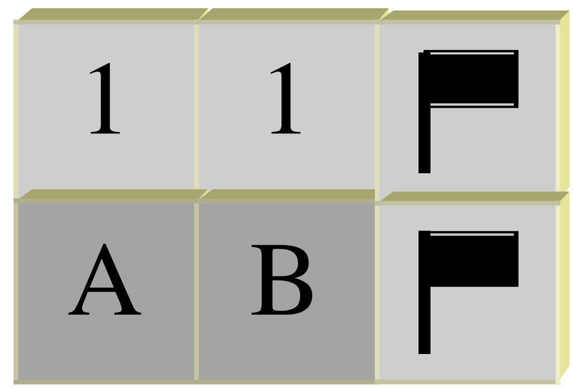
  + P(M(2, 1))

  - 0

  P(M(1, 1)) = P(M(2, 1))

  = .5

Mine
Sweeper



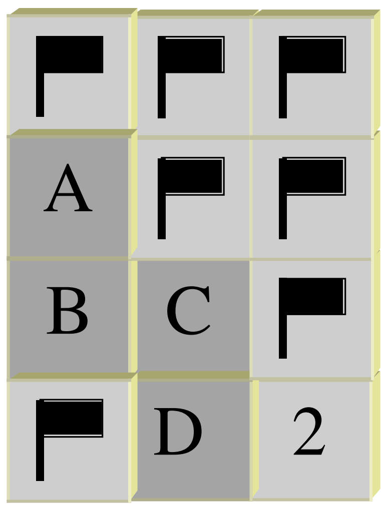
Joint Probability

- **The state of a single situation / domain may be represented by a *set* of random variables.**

Example of Joint Probability Distribution

Mine Sweeper

MinesLeft = 1



A	B	C	D	Probability
F	F	F	F	0
F	F	F	T	1/2
F	F	T	F	1/2
F	F	T	T	0
F	T	F	F	0
F	T	F	T	0
F	T	T	F	0
F	T	T	T	0
T	F	F	F	0
T	F	F	T	0
T	F	T	F	0
T	F	T	T	0
T	T	F	F	0
T	T	F	T	0
T	T	T	F	0
T	T	T	T	0