Lecture 16: Uncertainty

Prof. Louis Steinberg

Artificial Intelligence

CS 520: Introduction to
Progression

- Choose first step, what state does that result in?
- Plan from that state to goal

Means-Ends

- Choose first step, what state does that result in?
- Plan from the state to goal

Regression

- Choose last step, what conditions need to be true before it to
- Plan from the start to those conditions

Review: Building Plans

- Plan from start to choose conditions
- Ensure goal met after it?
- Plan from goal to start
Sources of Uncertainty

• Incomplete knowledge of the world state
  – Which room is the wumpus in?

• Uncertainty in operator effects
  – Command: turn right
  – Result: robot turns 89.35 degrees right

• Theoretical vs. practical uncertainty
  – It would cost too much to know.
  – We have no way to know, v.s.
    – Human time
    – Computer time
    – Human time
    – Computer time

• Incomplete knowledge of the world state
Modeling Uncertainty

- Degree of belief
- Historical interest: approximations of
- Oriented towards linguistic imprecision
- Fuzzy logic
- Probability

• Probabilistic
  - It is hot vs it is warm vs it is sweltering
Acting under uncertainty

- Probability and utility to decide what to do.
- Decision theory: A way to combine outcomes.
- Utility theory: a way to represent and reason about the desirability of different outcomes.
- Probabilistic methods for representing and reasoning about uncertainty.
- Bag with 10 marbles: 3 red, 7 blue
- $P(\text{red}) = 0.3$
- What fraction red? About 0.3
- Repeat lots of times.
- Reach in, take one, put it back

Probability
The probability for each value of a random variable if color = (red, blue)

\[ p(\text{color}) = (0.3, 0.7) \]

if color = (red, blue)

The probability for each value of a random variable.
Marbles are red or blue, striped or solid

Take marble, look at it, put it back

Ignore non-red

What fraction are striped?

P(red | striped) = 1/4

P(striped | red) = 1/3
Conditional Probability

\[
P(A | B) = \frac{P(A \cap B)}{P(B)}
\]

- Equivalently: \( P(A \cap B) = P(A | B) \cdot P(B) \)

\[
\begin{align*}
1 & = 3/4 = P(\text{red} | \text{striped}) \\
3 & = P(\text{red}) \\
1 & = 3 = P(\text{striped} | \text{red}) \\
1 & = 1/3 = P(\text{striped} | \text{red}) \\
4 & = 1/4 = P(\text{striped} | \text{red}) \\
1 & = 1/4 = P(\text{red} | \text{striped})
\end{align*}
\]
Basic Properties

• $0 \leq P(A) \leq 1$

• $P(true) = 1$

• $P(blue) = 0$

• $P(false) = 0$

• $P(black) = 0$

• $P(red) = 0$

• $P(green) = 1$

• $P(red \text{ and } blue) = 1$

$1 \geq P(\neg A) > 0$
\[ P(\text{red} \cap \text{striped}) = 0.6 \]

So subtract once

\[ P(\text{red} \cap \text{striped}) = P(\text{red}) + P(\text{striped}) - P(\text{red} \cap \text{striped}) \]

\[ P(\text{red} \cap \text{striped}) = 0.6 \]

\[ P(\text{red}) = 0.6 \]

\[ P(\text{striped}) = 0.4 \]

\[ P(\text{red} \cap \text{striped}) = 0.1 \]

\[ P(\text{red} \cap \text{striped}) = P(A \cap B) = P(A) + P(B) - P(A \cup B) \]

Basic Properties
Example Proofs

• Prove: \( P(true \mid A) = 1 \)
  
  Proof:
  \[
  P(true \mid A) = \frac{P(true \land A)}{P(A)} = \frac{P(A)}{P(A)} = 1
  \]

• Prove: \( P(B \mid A) + P(\neg B \mid A) = 1 \)
  
  Proof:
  \[
  P(B \mid A) + P(\neg B \mid A) = P(B \lor \neg B \mid A) + P(B \land \neg B \mid A) = P(true \mid A) + P(false \mid A) = 1 + 0
  \]
Example Proof

Prove: \( P(A) = P(A | B) \cdot P(B) + P(A | \neg B) \cdot P(\neg B) \)

Proof:

\[
\begin{align*}
P(B | A) + P(\neg B | A) &= P(B \lor \neg B | A) = P(\text{true} | A) = 1 \\
&= \frac{P(B | A) \cdot P(A) + P(\neg B | A) \cdot P(\neg A)}{P(A)} \\
&= \frac{P(A | B) \cdot P(B) + P(A | \neg B) \cdot P(\neg B)}{P(A)}
\end{align*}
\]

Example Proof
Example Use: Mine Sweeper

- Grid of cells, some have mines
- Probe a cell: 
  - If has a mine, lose 
  - Else tells number of neighboring mines

Variables:
- N: total mines
- C(x, y): neighbor mine count
- M(x, y): boolean: mine

Variables:
- Else tells number of neighboring mines
- If has a mine, lose

Example Use: Mine Sweeper
Mine Sweeper

\[ P(M(1,1)) = \frac{\text{# ways of choosing 2 of 5}}{6! / (3!3!)} = \frac{10/20}{1} = 0.5 \]

N = 3
P(C(1,2) = 2 \cap \sim M(1,2)) = \frac{\binom{3}{2} \cdot \binom{1}{2}}{\binom{6}{3}} = \frac{3 \cdot 2}{20} = \frac{3}{20} = 0.15

N = 3 = \text{Mine Sweeper}
\[
P(C(1,2) = 2 \land \neg M(1,2) \land M(1,1)) = \frac{\binom{3}{6} \cdot \binom{2}{1} \cdot \binom{2}{1}}{N} = \frac{2 \cdot 2}{20} = 0.2
\]
\[ P(C(1,2)=2 \land \sim M(1,2)) = P(C(1,2)=2 \land \sim M(1,2) \land M(1,1)) = \frac{2}{5} \]

\[ N = 3 \]

Mine Sweeper
\[ \frac{2/3}{0/3} = \frac{2/5 \times 1/2}{3/10} \]

\[ P(C(1,2)=2 \mid \overline{M}(1,2)) \]

\[ = \left( \frac{p(M(1,1)) \times p(C(1,2)=2 \mid \overline{M}(1,2))}{p(C(1,2)=2 \mid M(1,1)) \times p(M(1,1))} \right) \]

\[ = \frac{2/3}{3} = \frac{2}{N} \]

Mine Sweeper
Known: $P(M(1, 1) \lor M(2, 1)) = 1$
$P(M(1, 1) \land M(2, 1)) = 0$
$P(M(1, 1)) = P(M(2, 1))$

Conclude: $1 = P(M(1, 1)) + P(M(2, 1)) - 0$
$P(M(1, 1)) = P(M(2, 1)) = 0.5$

\[\begin{array}{ccc}
A & B & \text{Mine}\\
\text{Mine} & \text{Sweeper} & \\
\text{Sweeper} & & \end{array}\]
Joint Probability

• The state of a single situation / domain may be represented by a set of random variables.
### Example of Joint Probability Distribution

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**MinesLeft = 1**

**Sweeper**

**Mine Probability Distribution**

Example of Joint