

1. **[20 points]** The Farmer, Fox, Goose, and Grain problem is described as follows: A farmer wants to move himself, a silver fox, a fat goose, and some tasty grain across a river, from the west side to the east side. Unfortunately, his boat is so small he can take at most one of his possessions across on any trip. Worse yet, an unattended fox will eat a goose, and an unattended goose will eat grain, so the farmer must not leave the fox alone with the goose or the goose alone with the grain. What is he to do?
 - (a) Formulate the Farmer, Fox, Goose, and Grain problem in terms of State Space Search. Include an informal description of each of the following:
 - A representation for states.

A state is a tuple of 4 booleans, one for each of farmer, fox, goose, and grain, which is true if they are on the east side of the river and false otherwise
 - A set of operators, their state transitions and the applicability condition for each to avoid resulting in an illegal state (one in which something gets eaten). How many operators are needed?

Note: You need not describe every single operator in detail but your sample should be representative.

There are 4 operators: the farmer crosses alone or with one of his three possessions. In each case the precondition is that the fox and goose booleans not be the same unless one of them is being taken with the farmer, and the same for the goose and the grain. E.g., for farmer crosses alone, the precondition is $\text{goose} \neq \text{fox}$ and $\text{goose} \neq \text{grain}$.
 - A goal test.

All booleans are true
 - A non-trivial admissible heuristic evaluation function.

The number of false booleans divided by 2.

You should provide the best solutions you can think of.

2. [25 Points] Suppose you are building a knowledge-based system to plan the seating at a dinner party. You want to use the system to prove statements of the form $OK(P1, P2)$ (or $\neg OK(P1, P2)$) meaning it is (or is not) OK for person $P1$ to sit next to person $P2$. Suppose you give the system the following axioms:

$$\begin{aligned}
 &(\forall x, y) Dislikes(x, y) \rightarrow \neg OK(x, y) \\
 &(\forall x, y) Male(x) \wedge Male(y) \rightarrow \neg OK(x, y) \\
 &(\forall x, y) Female(x) \wedge Female(y) \rightarrow \neg OK(x, y) \\
 &(\forall x, y) \neg Dislikes(x, y) \wedge Male(x) \wedge Female(y) \rightarrow OK(x, y). \\
 &(\forall x, y) \neg Dislikes(x, y) \wedge Female(x) \wedge Male(y) \rightarrow OK(x, y). \\
 &Male(John) \wedge Female(Susan) \wedge Male(David) \wedge Female(Jane) \\
 &Dislikes(Susan, Dave) \wedge Dislikes(Jane, John)
 \end{aligned}$$

- (a) Do the axioms entail $OK(Susan, John)$? Explain.
 No, to conclude this we would have to have $\neg Dislikes(Susan, John)$
- (b) Do the axioms entail $\neg OK(Susan, John)$? Explain.
 No, to conclude this we would have to have $Dislikes(Susan, John)$
- (c) Do the axioms entail $OK(Susan, John)$ under the Closed World Assumption? Explain.
 Oops - we did not cover the Closed World Assumption. Ignore this question
- (e) Which of the answers (a,b and/or c) will change if we rewrite all the axioms by replacing each atom $Dislikes(...)$ with $\neg Likes(...)$? Explain.
 None (except c which we are ignoring)
- (a) [30 Points] Four criteria were defined for comparing search strategies: completeness, optimality, time complexity, and space complexity. Which one (1) of these four criteria **best** explains why:
- i. Iterative-Deepening search is usually preferred over Breadth-First search.
 Space complexity
 - ii. Iterative-Deepening search is usually preferred over Depth-First search.
 completeness
 - iii. Alpha-beta Minimax search is usually preferred over pure Minimax search.
 Time complexity
 - iv. A* search is usually preferred over Greedy search.
 optimality
 - v. Algorithm X might be preferred over A* search.
 space complexity
 - vi. Bidirectional search is usually preferred over Breadth-First search.
 time complexity

- (b) [20 Points] Definition: A heuristic function h is said to be monotone if it satisfies $[h(n) - h(n')]$ is less than or equal to $c(n, n')$ for all arcs (n, n') in the search graph where $c(n, n')$ is the cost of arc (n, n') .

Prove that if h is a monotone and $h(g)=0$ for all goal states then h must be admissible.

$$h(g)=0 \quad i = \text{distance}(g \text{ to } g)$$

Consider the set of nodes for which there is a path to g with at most j arcs. Suppose that for all such nodes n' $h(n') \leq \text{distance}(n', g)$. Let n' be such a node. Let n be a node not in this set but adjacent to one or more nodes in the set. Let n' be a node in the set and adjacent to n , and of all such nodes be the one on the cheapest path from n to g . Then $\text{distance}(g \text{ to } n) = \text{distance}(g \text{ to } n') + c(n, n')$
 $i = h(n') + [h(n) - h(n')] = h(n)$.

- (c) [20 Points] Given the following axioms:

$$(\forall u) \text{Last}(\text{cons}(u, \text{NIL}), u)$$

$$(\forall x, y, z) \text{Last}(y, z) \Rightarrow \text{Last}(\text{cons}(x, y), z)$$

- i. Prove the following theorem from these axioms by the method of Generalized

$$\text{Modus Ponens: } (\exists v) \text{Last}(\text{cons}(2, \text{cons}(1, \text{NIL})), v)$$

$$\text{goal: Last}(\text{cons}(2, \text{cons}(1, \text{NIL})), v)$$

$$\text{unify with Last}(\text{cons}(x, y), z). \quad x/2, y/(\text{cons}(1, \text{NIL})), z/v$$

$$\text{subgoal: Last}(\text{cons}(1, \text{NIL}), z)$$

$$\text{unify with Last}(\text{cons}(u, \text{NIL}), u) \quad u/1, z/u$$

- ii. Use answer extraction to find v , the last element of the list $(2, 1)$.

$$v/z, z/u, \text{ and } u/1 \text{ so } v=1$$

- (d) [15 Points] The Fibonacci numbers are defined by the following recurrence:

$$F(0) = 0,$$

$$F(1) = 1,$$

$$F(i) = F(i - 1) + F(i - 2) \quad i \geq 2.,$$

Write a **recursive** Lisp function called `Fibonacci` that takes one argument n and returns a list containing the Fibonacci numbers from $F(0)$ to $F(n)$ ordered from smaller to larger.

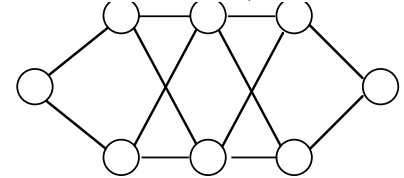
```
(defun fib (n)
  (cond ((= n 0) (list 0))
        ((= n 1) (list 0 1))
        (t (cons 0 (cons 1 (fib1 2 n 1 0))))))
```

```
(defun fib1 (n max fn1 fn2)
  (let ((fn (+ fn1 fn2)))
    (if (= max n) (list fn)
        (cons fn (fib1 (+ n 1) max fn fn1)))))
```

Notes:

- Your function (as well as any helping functions you may create) should be as time efficient as possible.
- You need not check for negative numbers or fractions
- You **must not** use the built in function *reverse* or any loop structures (like *dotimes* or *do* or *loop* ...etc.) in your solution.

3. [15 points] Here is a puzzle: In the figure below, suppose we wanted to number the nodes of the graph with numbers from 1 to 8 so that no two nodes have the same number and no two nodes that have an arc connecting them have consecutive numbers, (DON'T



actually number the nodes!)

- Formulate this puzzle in terms of State Space Search. Include an informal description of each of the following:
 - A representation for states.
a partially-labeled graph
 - A set of operators, their state transitions and the applicability condition for each to avoid resulting in an illegal state.
assign number to an unlabeled node. Precondition: no other node has this label nor do adjacent nodes have this label +/- 1
 - A goal test.
all nodes labeled
 - Would it be better to view this problem as a constraint satisfaction problem than a state space problem? Why or why not?
yes - it is a csp - eg we can assign labels in any order
4. [10 points] For each of the following statements, express it in First Order Predicate Calculus or explain why this cannot be done.

- All robots are artificial.
 $\forall r \text{ robot}(r) \Rightarrow \text{artificial}(r)$
- Not all robots are intelligent.
 $\neg \forall r \text{ robot}(r) \Rightarrow \text{intelligent}(r)$
- Everything that is artificial and intelligent is an AI.
 $\forall x \text{ artificial}(x) \wedge \text{intelligent}(x) \Rightarrow \text{AI}(x)$

- Every robot has at least one master.
 $\forall r \text{ robot}(r) \Rightarrow \exists m \text{ master}(m, r)$
- A robot that does not have a master is a dangerous.
 $\forall r (\text{robot}(r) \wedge \neg \exists m \text{ master}(m, r)) \Rightarrow \text{dangerous}(r)$

5. [20 points] Suppose we want to write a program for the following task: the program will be given a table of numbers, such as

x1	x2	x3	x4	y
1	1	2	3	0
2	3	5	1	10
1	7	4	2	9

and an expression such as

$$y = (f1 (f2 x1 x2) (f3 x3 x4))$$

where the x's are variables that hold real numbers, and the f's are functions. We want to find a set of definitions for these functions so that the expression as a whole computes the results given in the table, e.g. so that if x1 is 1, x2 is 1, x3 is 2, and x4 is 3, the value of the whole expression is 0. Each f must be defined to be one of +, *, -, or /. E.g., a solution for the specific problem given above is

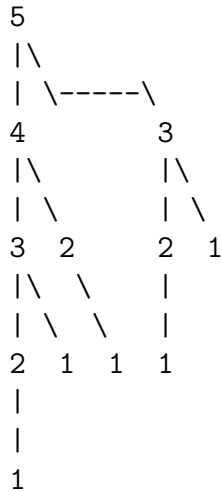
$$f1 = +, f2 = *, f3 = -$$

- Formulate this puzzle in terms of State Space Search. Include an informal description of each of the following:
 - A representation for states.
 partial assignment of ops to fs
 - A set of operators, their state transitions and the applicability condition (if any) for each to avoid resulting in an illegal state.
 add an assignment of an op to an f. if last op assigned, dont assign it so that a line of the table is wrong
 - A goal test.
 fs have ops
- Would it be better to view this problem as a constraint satisfaction problem than a state space problem? Why or why not?
 Yes, as in similar question above, though less so since we can only test at leaves

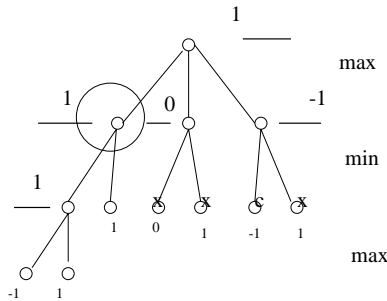
6. [15 points] Games.

- a. The game of nim is played as follows: You start by placing 5 pennies on the table. The players take turns removing pennies. At each turn, the player whose turn it is must remove 1 or 2 pennies from the table. The player who removes the last penny loses.

Draw the tree of game states.



The tree below is a tree of game states for some game. The leaves labeled with 1 are wins for the player who moves first, those labeled with -1 are losses for that player, and 0 are ties. Label the rest of the nodes with their values (in the spaces provided), and circle the node that indicates the best first move for that player to make.



- c. Mark an X through any node that would be pruned by alpha-beta pruning.

7. [20 points] LISP

Suppose we have a state-space search program written in Lisp, with the following definition for the structure representing a state:

```

;;; A state has 4 fields:
;;; parent is state that this state was reached from,
;;;      or nil if this is the start state
;;; op is the operator used to do so
;;; h is the state's heuristic value
;;; data is the data that represents the state
(defstruct state parent h data op)

```

Complete the following definition. Your code must use recursion and not iteration.

```

;;; creates a list of the operators used to reach the state
;;; end-state from the start state, in reverse order (so car
;;; of the list is the operator that created end-state)

```

```

(defun op-list (end-state)
  (if (null end-state) nil
      (cons (state-op end-state)
            (op-list (state-parent end-state)))))

```

)

8. [25 points] For each of the following statements, express it in First Order Predicate Calculus or explain why this cannot be done.

- Every professor teaches at least one class.
 $\forall p \text{ professor}(p) \Rightarrow \exists c \text{ class}(c) \wedge \text{teaches}(p, c)$
- Every student likes at least one class that he takes.
 $\forall s \text{ student}(s) \Rightarrow \exists c \text{ class}(c) \wedge \text{takes}(s, c) \wedge \text{likes}(s, c)$
- Freshmen typically take the class CS111.
 Cannot be done - can't express "typically" in FOL
- No class has both freshmen and seniors taking it.
 $\neg \exists c \text{ class}(c) \wedge \exists f \text{ freshman}(f) \wedge \text{takes}(f, c) \wedge \exists s \text{ senior}(s) \wedge \text{takes}(s, c)$
- Every student who is taking a class taught by Prof. Steinberg is also taking a class taught by Prof. Kaplan.
 $\forall s (\text{student}(s) \wedge \exists c \text{ course}(c) \wedge \text{teaches}(\text{Steinberg}, c) \wedge \text{takes}(s, c)) \Rightarrow \exists c' \text{ course}(c') \wedge \text{teaches}(\text{Kaplan}, c') \wedge \text{takes}(s, c')$
- Everything that is true for all birds is true for all eagles.
 Can't do in FOL; requires quantifiers on predicates.

- If you are in a room next to the room the wumpus is in, the effect of waving a magic wand is that the wumpus will move into the room you are in.

$$\forall a \forall r \forall r' (in(agent, r, s) \wedge in(wumpus, r', s) \wedge nextto(r, r')) \Rightarrow in(wumpus, r, resultof(wavewand, s))$$

- If the weather report predicts rain, then 90% of the time it will rain. Can't do - FOL does not handle probability.

9. [10 points] Convert each of the following to Conjunctive Normal Form.

- $\forall d (dog(d) \Rightarrow \exists p (person(p) \wedge owns(p, d)))$
 $\forall d (\neg dog(d) \vee \exists p (person(p) \wedge owns(p, d)))$
 $\forall d (\neg dog(d) \vee (person(s(d)) \wedge owns(s(d), d)))$
 $(\neg dog(d) \vee person(s(d))) \wedge (\neg dog(d) \vee owns(s(d), d))$
- $\exists d (dog(d) \wedge \forall f (dogfood(f) \Rightarrow eats(d, f)))$
 $\exists d (dog(d) \wedge \forall f (\neg dogfood(f) \vee eats(d, f)))$
 $(dog(s) \wedge \forall f (\neg dogfood(f) \vee eats(s, f)))$
 $(dog(s) \wedge (\neg dogfood(f) \vee eats(s, f)))$
 $(dog(s) \wedge \neg dogfood(f)) \vee (dog(s) \wedge eats(s, f))$

10. [20 points] Given the following clauses as the knowledge base, show a proof by resolution refutation that cheese(Moon):

$\neg large(x) \vee \neg green(x) \vee cheese(x)$
 $large(x) \vee \neg p(y) \vee \neg orbit(x, y)$
 $\neg rock(x) \vee green(x)$
 $p(Earth)$
 $orbit(Moon, Earth)$
 $orbit(Earth, Sun)$
 $rock(Moon)$

Note the typo - $p(y)$ was $p(x)$, which makes the problem impossible. I apologize for the confusion.

$\neg large(x) \vee \neg green(x) \vee cheese(x), \neg cheese(moon) \rightarrow \neg large(moon) \vee \neg green(moon)$
 $\neg large(moon) \vee \neg green(moon), \neg rock(x) \vee green(x) \rightarrow \neg rock(moon) \vee \neg large(moon)$
 $\neg rock(moon) \vee \neg large(moon), large(x) \vee \neg p(y) \vee \neg orbit(x, y) \rightarrow \neg rock(moon) \vee \neg p(y) \vee \neg orbit(moon, y)$
 $\neg rock(moon) \vee \neg p(y) \vee \neg orbit(moon, y), rock(moon) \rightarrow \neg p(y) \vee \neg orbit(moon, y)$
 $\neg p(y) \vee \neg orbit(moon, y), p(Earth) \rightarrow \neg orbit(moon, earth)$
 $\neg orbit(moon, earth), orbit(moon, earth) \rightarrow false$