# Stability Guaranteed MPC for Mobile Robot Navigation (Extended Abstract)

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## Introduction

A mobile robot needs to navigate in dynamic and unstructured environments. For this, motion planning algorithms must be able to handle complex, time-varying constraints presented by the environment, and quickly generate high quality trajectories that reaches the goal.

Model Predictive Control (MPC) is a receding-horizon control algorithm which optimizes the performance of the constrained systems on-line. (See (Rawlings 2000) (Mayne et al. 2000) (Lee 2011) for excellent reviews.) This can be a very useful tool for mobile robot navigation in dynamic environments due to its dynamic replanning framework, its ability to handle complex time-varying constraints, and its flexible formulation which allows the user to tune the behavior of the system to desired performance. Thus the MPC and MPC-like dynamic replanning framework are becoming more popular in robotics community, e.g. (Green and Kelly 2007) (Howard, Green, and Kelly 2009) (Dolgov et al. 2010) (Knepper and Mason 2012) (Du Toit and Burdick 2012) (Park, Johnson, and Kuipers 2012). However, it can be difficult to ensure stability and temporal consistency of solutions, especially for systems with differential constraints.

In this paper, we show that it is possible to provide a stability guarantee for MPC for mobile robots that can be modeled as unicycles (e.g. differentially driven carts) with a proper estimate of the cost-to-go. This critically depends on our non-holonomic distance function, which is also a control-Lyapunov function for unicycle-type vehicles. This allows us to properly estimate the cost-to-go for unicycle-type vehicles, and ensure convergence via MPC. We show theoretical and numerical analysis on the stability conditions.

## The Cost-to-go and The Stability Guarantee

Consider the standard cost formulation in discrete-time MPC that maps a trajectory of states $q_{[k:k+N]}$ and inputs $u_{[k:k+N-1]}$ to a cost

$$J(\cdot) = \sum_{i=k}^{k+N-1} L(q_i, u_i) + M(q_{k+N}) \quad (1)$$

where $L(q_i, u_i)$ is the *stage cost* computed along the trajectory, and $M(q_{k+N})$ is the *terminal cost* at the *terminal state* $q_{k+N}$. This terminal cost is often formulated as a cost-to-go from the terminal state to the eventual goal state, and a proper terminal cost is essential for the stability of MPC (Mayne et al. 2000).

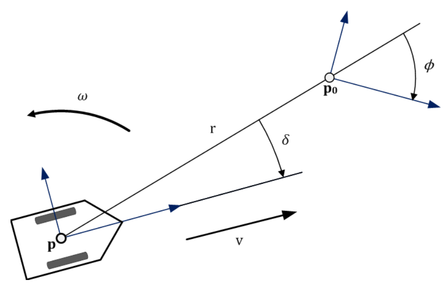


Figure 1: Egocentric polar coordinate system with respect to the vehicle. From an observer is situated on a vehicle and fixating at target pose $p_0$, $r$ is the radial distance to the target, $\theta$ is the orientation of $p_0$ with respect to the line of sight from the observer to the target, and $\delta$ is the orientation of the vehicle heading with respect to the line of sight. This coordinate system allows much simpler and intuitive computation of the cost-to-go and the control law in our formulation.

Following the idea introduced in (Jadbabaie, Yu, and Hauser 2001), we show that the origin is stable if the terminal cost is a control-Lyapunov function for the kinematics of the system. Specifically, we have

$$\begin{aligned} M(\cdot) &= M(r, \phi, \delta) \\ &= \sqrt{r^2 + k_\phi^2 \phi^2} + k_\delta |\delta - \arctan(-k_\phi \phi)| \end{aligned} \quad (2)$$

as our non-holonomic distance function which qualifies as a control-Lyapunov function (Park and Kuipers 2015) from a pose p to a target pose $p_0$, expressed in egocentric polar coordinates (Fig. 1). Our results can be used to guarantee that a robot can always move toward a specified goal or a waypoint along a path. This in turn guarantees the robot will reach the final destination at the end of the path that consists of a series of waypoints, constructed by existing path planners such as *RRT** (Karaman and Frazzoli 2011) and their non-holonomic variant (Karaman and Frazzoli 2013) (Park and Kuipers 2015).

Stability of the origin for the full dynamics of the system is also evaluated numerically (Fig. 2) using high-fidelity

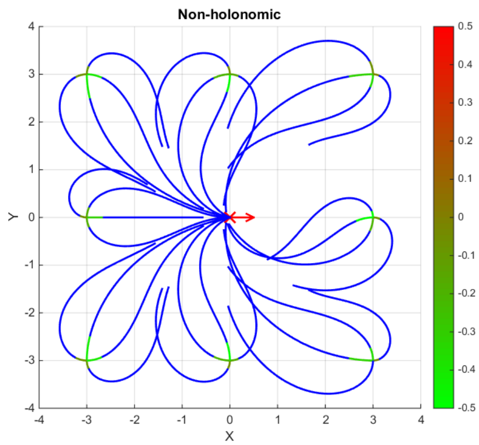


Figure 2: (Best viewed in color.) Example solutions from the MPC with the non-holonomic cost-to-go (3). The blue trajectories are optimal solutions obtained by 5 second horizon MPC with various initial configurations, all heading toward a target pose (red arrow) facing right. The change in the cost-to-go to the target in the initial segment (1 second) is color coded, where decrease in the cost-to-go is colored green and the increase is colored red. For stability, the cost-to-go needs to decrease in the initial segment everywhere, which is satisfied for all the 32 examples shown. The choice of the cost-to-go is critical. We note that this test readily fails when Euclidean distance is used, which is a popular choice in existing literature.

physics simulator of a wheelchair robot. Here, the control inputs are parameterized with a low-level control law (Park and Kuipers 2011) to reduce dimensionality of the optimization problem. We discuss the effect of control parameterization, initial conditions, and uncertainties in the environmental variables to the stability guarantee.

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