Zero-Control Linearization-Based Steering for Sample-Based Planning of Dynamic Systems

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We investigate steering and distance computations based on linearizations about zero-control trajectories for RRT-like motion planning of highly dynamic systems. With planning constrained to dynamically feasible trajectories, the concept of "straight" is no longer Euclidean but instead determined as the path with least cost to transfer from one state to another. This additional complexity transforms steering and nearest neighbor computations into optimal control problems for which numerical techniques are slow compared to solving Euclidean distance. However, the optimal control solutions can be approximated as Linear Quadratic Regulation or Tracking problem (LQR or LQT) through linearizations of the cost and dynamics as done in Glassman and Tedrake [2010]; Goretkin et al. [2013]; Perez et al. [2012]; Webb and van den Berg [2013], who all linearize about a single point—e.g. an explored state or a sampled state. In comparison, we propose linearizing about the zero-control trajectories. The advantage here is that the approximation remains valid for longer time-horizons at the expense of solving time-varying LQR instead of time-invariant LQR.

This abstract and the subsequent interactive session workshop compares zero-control linearization and single point linearization distance based computations for RRT planning of a pendulum on a cart system. The system has four states which are the cart position, cart velocity, pendulum angle, and pendulum angular velocity. The example system is shown in Figure 1 along with trajectories explored by an RRT execution with a tree of 1000 vertices for differing max time horizons. As is qualitatively apparent from the figure, planning with longer time horizons results in a denser explored space for an equal number of vertices. This statement is quantified in Figure 2 which compares the number of vertices and execution time to discover a state within $\delta = 2$ away from x_{goal} marked by the green star in Figure 1. Note that the start and goal states are upright, have zero velocity and start 6 meters apart. The results are discussed further later.

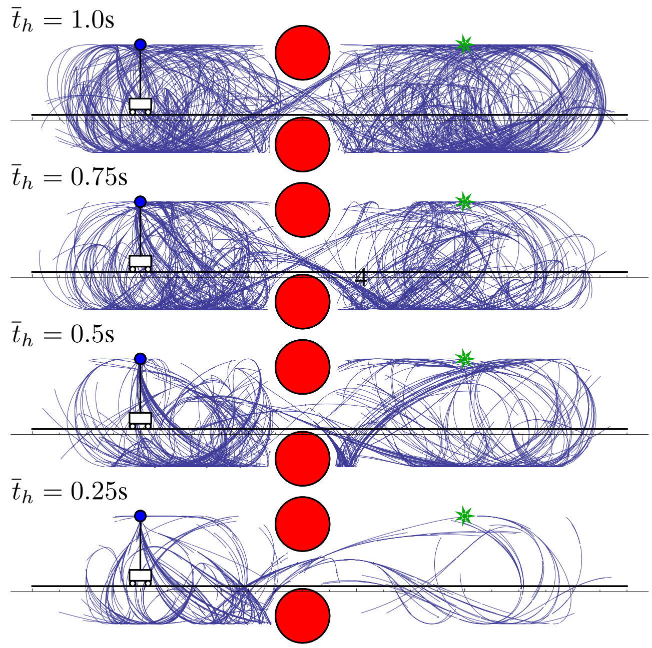

Figure 1: Shows the 1000 explored trajectories of pendulum on a cart example for executions of RRT with differing max time horizons. The initial state of the pendulum is shown along with two obstacles and the goal state x_{goal}. Note that the trajectory velocities are not directly reflected and so trajectories appearing to be near x_{goal} may be high velocity.

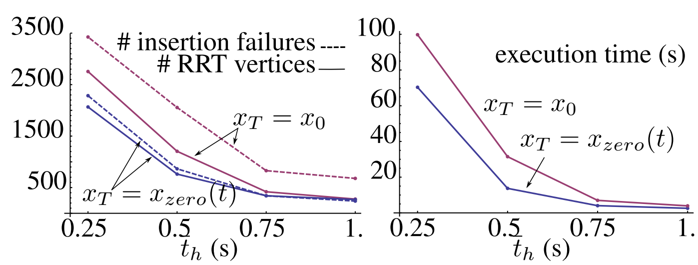

Figure 2: Comparison of average number of RRT vertices, average number of insertion failures and average execution time for 100 runs of RRT at differing max time horizons. We stop the RRT once a state $x' \in X$ was $\|x' - x_{goal}\| < \delta$ where δ was chosen to be 2.

Planning Review

We quickly review planning for dynamic systems. The kinodynamic planning problem is to find a state and control trajectory (x, u) that connects a start state x_{start} to a goal state x_{goal} that satisfies the differential constraint given by the system dynamics

$$\dot{x}(t) = f(x(t), u(t)). \quad (1)$$

If a trajectory satisfies the system dynamics, then we say it is feasible. Sample-based planning algorithms find a path by generating a graph $G = (V, \mathcal{E})$ where the vertices V are explored states $x \in X$ and the edges $\mathcal{E}$ are state and control trajectories that connect two states in V—i.e. $(x, u; t_h) \in \mathcal{E}$ implies for time horizon $t_h > 0$, $x(0) \in V$, and $x(t_h) \in V$. Every edge must be dynamically feasible in that the state and control must satisfy Eq. 1. Additionally, the state and control must remain in the set of *allowable state and control* (X, U) throughout the full trajectory where the allowable state subset $X \subset \mathbb{R}^n$ and control subset $U \subset \mathbb{R}^m$. The set X is free of all obstacles.

Methods based on RRT build the graph through two functions: nearest neighbor and steering. The nearest neighbor calculation relies on the choice of distance function between a state $x_0 \in V$ and the sampled state $x_{samp} \in X$. Ideally, this distance is the cost J to transfer x_0 to x_{samp}. Then, the nearest state is the $x_0 \in V$ with least distance to x_{samp}. Steering computes a trajectory that satisfies Eq. 1 and transfers the system from an initial state $x_0 \in V$ to a neighborhood of a desired state $x_{samp} \in X$. Such a problem can be treated as an optimal control problem which finds a trajectory that minimizes some cost J.

The cost for computing distance and steering can be the same. We consider the following cost function:

$$\begin{aligned} J(x, u; t_h) := \frac{1}{2}\int_0^{t_h} u^T(\tau)R(\tau)u(\tau)\, d\tau \\ +\tfrac{1}{2}(x(t_h) - x_{samp})^T P_1 (x(t_h) - x_{samp}) \end{aligned} \quad (2)$$

where $R = R > 0$ is symmetric positive definite and $P_1 = P_1^T \geq 0$ is symmetric positive semi-definite. A more general form can be taken but is outside the scope of this abstract.

Linearization-Based Steering Review

The problem of minimizing J constrained to the dynamics Eq. 1 is a nonlinear optimization problem for which numerical methods can be slow. For this reason, we approximate by linearizing about one of two candidates. The candidate used in Glassman and Tedrake [2010]; Goretkin et al. [2013]; Perez et al. [2012]; Webb and van den Berg [2013] is simply the point x_0. The second candidate, which we propose, is the zero-control trajectory $x_{zero}(t)$ and is the solution to:

$$\dot{x}_{zero}(t) = f(x_{zero}(t), 0), \text{ s.t. } x_{zero}(0) = x_0.$$

Let $x_T(t) = x_0$ or $x_{zero}(t)$ depending on the candidate chosen. The linear terms are $A(t) = \frac{\partial}{\partial x(t)} f(x(t), u(t))|_{(x_T(t),0)}$ and $B(t) = \frac{\partial}{\partial u(t)} f(x(t), u(t))|_{(x_T(t),0)}$ with approximate state and control

$$\begin{aligned} \tilde{x}(t) = x_T(t) + z(t), \text{ and } \tilde{u}(t) = v(t), \\ \text{where } \dot{z} = A(t)z(t) + B(t)v(t), \text{ s.t. } z(0) = 0. \end{aligned}$$

Using the linearization, steering and distance computations are LQT problems, which we solve using our Algorithm 4: Efficient Fixed t_h Inexact Linear Steering in Caldwell and Correll [2015], which is similar to the approach in Goretkin et al. [2013].

For steering, since the approximate state and control $(\tilde{x}, \tilde{u})$ cannot be a feasible trajectory unless Eq. 1 is linear, we project the approximate state and control onto the space of feasible states and control through

$$\begin{bmatrix} x \\ u \end{bmatrix} = \mathcal{P}\left(\begin{bmatrix} \tilde{x} \\ \tilde{u} \end{bmatrix}\right) := \begin{cases} \dot{x} = f(x, u) \\ u = \tilde{u} - K(x - \tilde{x}). \end{cases} .$$

If the projected state and control (x, u) are admissible—e.g. do not collide with an obstacle—then they constitute a new edge insertion to the tree. If they are inadmissible, we say there was an *insertion failure*.

Results

For max time horizons $\bar{t}_h = 0.25, 0.5, 0.75, 1.0$s, we execute the RRT 100 times. An execution is stopped once a state $x' \in X$ is discovered where $\|x' - x_{goal}\| < \delta = 2$. The results for the average execution time, average number of vertices, and average number of insertion failures is shown in Figure 2. It is evident that the number of vertices required reduces significantly for the greater time horizons, which is reflected in the execution time. Comparing the results for linearizing about x_0 and $x_{zero}(t)$, we find that the number of required vertices is nearly the same, but the number of insertion failures is significantly greater for x_0, which is also reflected in the execution time. This increase in the number of insertion failures is due to the quality of the approximation.

While implementations that better handle the time invariance of linearizing about x_0 may alleviate the execution time of additional insertion failures for the pendulum example, the number of insertion failures will be protracted for systems with more extreme nonlinearities.

References

T.M. Caldwell and N. Correll. Long time horizon steering for sample-based planning of dynamic systems. 2015. URL http://timbot2.com/long-time-horizon-planning/. (In Preparation).

E. Glassman and R. Tedrake. A quadratic regulator-based heuristic for rapidly exploring state space. *IEEE International Conference on Robotics and Automation*, pages 5021–5028, 2010.

G. Goretkin, A. Perez, R. Platt Jr., and G. Konidaris. Optimal sampling-based planning for linear-quadratic kinodynamic systems. *IEEE International Conference on Robotics and Automation*, pages 2429–2436, 2013.

A. Perez, R. Platt, G. Konidaris, L. Kaelbling, and T. Lozano-Perez. LQR-RRT*: Optimal sampling-based motion planning with automatically derived extension heuristics. *IEEE International Conference on Robotics and Automation*, pages 2537 – 2542, 2012.

D. J. Webb and J. van den Berg. Kinodynamic RRT*: Asymptotically optimal motion planning for robots with linear dynamics. *IEEE International Conference on Robotics and Automation*, pages 5054–5061, 2013.