Visualization via Polynomiography and Application in CS, Math and Art

CS Topics 198:442, Index: 28588, Semester: Fall 2013, Credits: 4, Recitation

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Lecture: T4 1:40-3:00 PM, Hill 250 - Busch
Lecture: Th4 1:40-3:00 PM, Hill 120 - Busch
Recitation: T 12:15-1:10 PM, Hill 250 - Busch

Course Description

Polynomiography is the algorithmic visualization of the process of polynomial root-finding, giving rise to diversely interesting and interdisciplinary applications in science, math, education and art. While a real or complex polynomial equation is nothing but an algebraic encryption of a set of points in the Euclidean plane, polynomials are present in every branch and mathematics and the sciences, and are building blocks at all levels of formal education. Visualization of polynomials via polynomiography can also turn polynomials into fundamental or desirable tools of art. Surprisingly, there are seldom courses solely dedicated to the study of polynomials. By considering the visualization of polynomials via polynomiography, this course offers a novel and modern point of view into theoretical, algorithmic and practical properties pertaining to polynomials and their applications in computer science, mathematics, education, art and more. Polynomiography gives rise to many interesting interdisciplinary applications. In this course students will have the opportunity of working with a polynomiography demo software. Using polynomiography software, as well as existing packages, the course will study a subset of topics on polynomials and polynomiography and their applications, such as:

- Geometric Properties of Complex Polynomials
- Algorithms for the Computation of Polynomial Roots and Their Polynomiography
- Local and Global Behavior of Iteration Functions, e.g. Newton's Method
- Topics in Dynamical Systems: Fatou and Julia sets, Fractals, Mandelbrot Set
- Negative Results On Solving Equations: Unsolvability, Undecidability, Non-convergence
- Connections to Computational Geometry, e.g. Voronoi Diagrams
- Connections to Combinatorics and Graph Theory
- Polynomiography and Computer Graphics
- Quaternion Polynomiography and Computer Graphics
- Visual Cryptography and Polynomiography
- Educational Applications: Visualization of Math Concepts and Computer Algorithms
- Fine Art: Such as Polynomiography-Inspired Painting, Sculpting, Dance
- Polynomiography-Inspired Smartphone Applications, Games.

The study of these topics will result in fundamentally new applications and appreciation of polynomials, leading to new and worthy research topics of various kinds in computer science,
mathematics, education, and other fields. We will offer new geometric and algorithmic results on polynomials and polynomial root-finding, e.g. visually appealing proofs of classical results such as the fundamental theorem of algebra, the maximum modulus principle, the Gauss-Lucas theorem, interpretations of Newton’s method, as well as a novel polynomial root-finding algorithm that make use the ellipsoid method, giving rise to their own polynomiography. We will also study some results from computational geometry that are relevant to polynomiography. Another subject of study is algorithms for quaternion polynomial root-finding, applications, connections to complex polynomial root-finding, 2D and 3D quaternion polynomiography.

Audience: Students in computer science, also math and other majors. While some of the underlying concepts are advanced, the goal is to make these concepts understandable to students with little or no background in the related topics. For instance, while we will study complex polynomials, no knowledge of a complex analysis will be needed. The course is to be self-contained in the sense that the selected topics of study will be chosen from the set of references listed below and some articles referenced therein. No textbook is needed, however one or more books will be placed on reserve. Handouts will be provided on regular basis.

Prerequisite: 01:198:112, 01:198:205, and 01:198:206, or permission of instructor. Knowledge of Java, C, or Python is useful. Background in algorithms, and linear algebra. Students who choose to do a programming project can do the implementation in the programming language of their choice.

Grading: Students will be asked to do a class project, selected according to their interest. However, it must be a novel project that is inspired by the course material, also requiring approval of the instructor. The project will count 45 percent of the grade and it will be either theoretical, or practical. A theoretical project may consist of a mathematical or algorithmic work. A practical project may consist of computational implementation of an algorithm and computational results; implementation of an animation; creation of artistic work; a Smartphone application; a game, etc. Several assignments will be given, counting for another 45 percent. The remaining 10 percent of the grade will be based on class participation.

References


