

CS 521: LINEAR PROGRAMMING, Fall 2017, 3 Credits

INSTRUCTOR: Bahman Kalantari (kalantar@cs.rutgers.edu)

LECTURE: Thursday 5:00-8:00 PM, LSH-B267, LIV (Livingston).

OFFICE HOURS: Wed 12:00-1:00 PM, Hill Center 444 (also by appointment).

TA: Diana Kim, dsk101@scarletmail.rutgers.edu, CBIM, near 03, Office Hrs: Friday 11:00am-12:30 pm.

PREREQUISITES: DCS graduate study admission requirements or permission of Instructor (Elements of Linear Algebra, Calculus and Multivariable Calculus).

GRADING: The better of (I) and (II); or Option (III).

(I) HWKS (4 written, a MATLAB Program) %30; Midterm %30; Final %40.

(II) HWKS (4 written, a MATLAB Program) %30; Final %70.

(III) HWKS (4 written) %20; Midterm %20, Final %20; A Substantial MATLAB Program %40.

All programming assignments must be discussed and approved by the instructor.

LECTURE DATES: Sept 7, 14, 21, 28; Oct 5, 12, 19, 26; Nov 2, 9, 16, 21 (Tue), 30; Dec 7.

EXAM DATES: MIDTERM: October 19, FINAL: December 14.

COURSE OUTLINE:

- The *convex hull membership problem* (CHM) and its generalization: An introduction to *linear programming* (LP) and *support vector machine* (SVM) problems. A *distance duality* and the *triangle algorithm*, a geometric algorithm for CHM and SVM.
- Linear inequalities and the *feasibility problem*. The linear programming problem (LP). Related formulations of LP: The *standard form*, the strict feasibility problem.
- Dantzig's *simplex method*. Starting the simplex method, *Basic Feasible Solutions*. Phase I and Phase II methods. Degeneracy, cycling. *Bland's rule* for finite termination. Worst-case complexity of LP: *Klee-Minty* examples. *Hirsh conjecture*. Sensitivity analysis, parametric LP.
- *Farkas lemma*, *Gordan theorem*, geometric interpretations. Algorithmic applications.
- Duality in linear programming. *Lagrange multipliers* and *Karush-Kuhn-Tucker* optimality conditions. Duality theorems. *Complementary slackness conditions*. *The fundamental theorem of LP*. Dual of LP in Standard form and its corresponding complementary slackness conditions.
- The *dual simplex method*. The *primal-dual method* for LP and some applications.
- Convex sets, Polyhedra and polyhedral cones, extreme points, extreme directions, recession directions, edges, facets, basic feasible solutions. *Fourier-Motzkin elimination method*, its worst-case complexity and applications. Representation theorems: *Caratheodory*, *Farkas-Minkowski-Weyl*, and *Helly* theorems.

- *Game theory* and *von Neumann's min-max theorem*.
- The triangle algorithm: *A fully polynomial-time approximation scheme* for CHM and for algorithmic *separation of convex sets*. The particular case of SVM.
- *Khachiyan's ellipsoid method* for LP. Connections to CHM. Notions of size of LP, rounding, precision, and *polynomial-time algorithms*.
- *Karmarkar's algorithm* and variations, connections to CHM.
- Some properties of convex functions. *Taylor theorem*.
- The positive semidefinite *diagonal matrix scaling* problem, connections to CHM. *Scaling dualities*. Polynomial time *potential-reduction* and *path-following Newton methods* for matrix scaling/linear programming problems.
- *Strongly polynomial-time algorithms*. *Total unimodularity* and *structured LP*. Topics from: shortest paths, mean cycles, max flows, bipartite matching, min-cost flows, multicommodity flows, minimum spanning tree, general weighted matching problem, TSP, and magic labeling problem.

TEXT: Lecture notes will be made available. Also some literature articles.

OTHER REFERENCES (to be placed on reserve at Math Library, Hill Center

Linear Programming, Chvátal, Freeman and Company, 1983.

Introduction to Linear Optimization by Bertsimas & Tsitsiklis, Athena Scientific, 1997)

Bazaraa, Jarvis and Sherali, Linear Programming and Network Flows, Wiley, 1990.

Schrijver, Theory of Linear and Integer Programming, Wiley, 1986.

Papadimitriou and Steiglitz, Combinatorial Optimization: Algorithms and Complexity, Prentice-Hall, 1982.