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THE PROBLEM OF DIAGNOSIS WITH RESPECT TO PHYSICAL PARAMETERS FOR CHANGES IN STRUCTURES.

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ABSTRACT

Assuming that a structure is modeled by a tripyle M.E.C. (mass elasticity and damping matrix), once a detection of a change is signaled, we try to answer the diagnostic problem, i.e. what is the most probable variation in the physical parameters that caused the alarm. The diagnosis method is based on the Instrumental Statistic Detection Method. It basically consists in relating the statistic into certain precomputed subspaces that have a known correspondence with the physical parameters. The subspace that maximizes the statistic is considered as the most probable cause of the signaled change.

RESUME

En supposant que une structure est modelee par un triplet M.E.C. (matrices de masse, elasticite et amortissement) une fois que une detection de rupture est signalee, on essai de responder au probleme de diagnostic, i.e. qu'est ce qui est varie dans les parametres physiques qui a cause l'alarme. La methode de diagnostic est basee sur la Methode de Detection des Variables Instrumentales. Elle consiste a maximiser la statistique dans des sous-espaces prevus qui ont une correspondance directe avec les parametres physiques. Le sous-espace qui maximise la statistique est considere comme la cause la plus probable du changement signalé.
1. INTRODUCTION.

Let us assume that a structure is described by the following differential equation

\[ M \ddot{X} + C \dot{X} + KX = F(t) \]

\[ Y = LX \]

where \( M, C, K \) are the mass, damping and stiffness matrices, \( X \) is the state vector, \( F(t) \) the input and \( Y \) the output (response). For the three matrices \( M, C, K \) we will assume that they are symmetric but the mass matrix is not necessarily diagonal. A linear model of the form of (1) is very common for modeling structures even for cases where partial differential equations are needed. It can be for example the model generated by the Finite Element Method (FEM). Usually the matrices \( M, C, K \) generated by FEM have a large dimension (for practical problems). If we try to estimate the natural frequencies using the FEM model, the estimation is good only for the first two or three modes. On the other hand the \( M, C, K \) representation has a very direct correspondence with the physical structure, we can easily identify the structure by using the elements of these matrices. For this last reason the \( M, C, K \) representation is tractable for the diagnostic problem.

If a large structure is vibrating under excitation and we measure these vibrations we can very seldomly correctly identify by any method more than a very limited number of modes. This is because higher modes are not excited enough and are thus lost in the measurement noise. Thus an \( M, C, K \) model even though tractable for a diagnosis, is too complicated for a detection of changes in vibrations, where a more accurate but reduced representation of the relevant modes is needed. For such a case a reduced AR model is more efficient. It has the advantage that it can be estimated using classical estimation methods.
directly on output measurements, something that is not possible for the M.K.C. model, where the costly PEM is used. In the next section we will talk briefly about the identification problem and try to relate the AR model with the M.K.C. model.

II. THE IDENTIFICATION OF A REDUCED MODEL

Let us assume that we have available a collection of samples \( y_1, \ldots, y_T \) of the output \( y \) of a system under normal conditions and also a reduced model of the form of (1), i.e., the matrices M.K.C. What we would like to do is to estimate an AR model, that is a set of matrices \( A = [A_0, \ldots, A_m] \) such that \( Y \) defined by

\[
Y = Y_0 - A_0 Y_0 - \cdots - A_m Y_0
\]

behaves as a moving average sequence. The AR model defined in (2) has an order which is much smaller than the order of the model in (1), because it contains only the excited modes. Once we have selected an order, there exist several methods for estimating \( A \) using the output samples. See for example [12]. Let us call \( A_{(n)} \) the AR model obtained by using the output samples. This model identifies correctly the excited modes and is usually sufficient to describe the statistical behavior of the output signal. This is the reason why it is used for detection problems.

Let us now consider the estimation of \( A \) by using the model in (1). It is clear that the sampled versions of the output \( y \) of (1) can be represented as an ARMA process. The problem is that the AR part will be too large since it contains all the modes of (1) and not only those that are excited. We thus need a model reduction. Let us assume that we have made a selection of eigenvalues and their corresponding eigenvectors. They are usually in complex conjugate pairs if \( (\lambda_1, \ldots, \lambda_{2m}) \) and \( (\rho_1, \ldots, \rho_{2m}) \) are the selected eigenvalues and their corresponding eigenvectors, we will assume throughout this paper that all the selected eigenvalues are simple. For a pair \( \lambda_k, \rho_k \) we have

\[
\{ X_k W + \lambda_k C + K \} \rho_k = 0
\]

Also because of the symmetry of the matrices we have

\[
\lambda_k (X_k W + \lambda_k C + K) = 0
\]

for \( k = 1, \ldots, 2m \). It is now easy to see that if \( \lambda_k \) are the only excited modes then the reduced AR model satisfies

\[
\{ mdt \rho_k - dh \rho_k - \cdots - \rho_k \} m k = 0
\]

where \( m_k = \exp(\lambda_k t) \) and \( t \) is the sampling period. If \( d \) is the length of the output vector \( y \) then \( d m_k \) must be equal to \( 2m \); if this is not possible we take \( m \) larger in order \( d m_k \) just exceeds \( 2N \). The system of equations defined in (5) can be put under a more compact form

\[
X_k W - A_k X_k = 0 \quad \cdots \quad - A_k X_k = 0
\]

where \( W = [0 \ 0] \) and \( X = [X_k X_k \ldots X_k] \)

\[
D = [D \ 0]
\]

\[
D = [0 \ D]
\]

is a diagonal matrix containing only half of the selected eigenvalues and \( D \) contains the conjugate eigenvalues. \( X \) is a matrix that contains half of the selected eigenvalues and \( X \) the corresponding eigenvectors. Clearly the eigenvectors in \( \psi \) correspond to the eigenvalues in \( \lambda \). If we now call \( E_k = \psi_k \psi_k^T \) then (5) can be written

\[
E_k = B A^T
\]

where \( B = [B_{m_k}] \). We call the matrix \( B \) identified using Equation (7).
A. Clearly $A_0$ and $A_1$ are not the same, because as we said in the introduction the FEM is not very accurate for the identification of the modes and also because the identification of $A_1$ is usually biased because of the model reduction. The only thing one should take care of in order to remove somehow the difference, is to select those eigenfrequencies from the M.E.C model that have frequencies close to the ones identified by using the $A_0$ model. In any case the difference between $A_0$ and $A_1$ is very crucial basically because $A_1$ is not used for any test. As we will see, it is used only for computing the different subspaces that are necessary for the diagnosis. In the next section we present the Instrumental Statistics Detection Method (ISDM) and how it can be used to make a diagnosis.

III. THE INSTRUMENTAL STATISTICS DETECTION METHOD

We briefly present this detection method by mating more in the property of applying it on a certain subspace. More details regarding theoretical questions and also practical implementation of the test can be found in [3,4,5]. Its optimality properties with maximization of its power by optimally positioning the sensor on the state vector can be found in [6,7].

Let we assume that we are given a nominal AR model $A_0$ and a collection of covariances $R_k E_m$ where $R_k = \sum_{i=1}^{N} X_i^2 E_i^2$ is the sample covariance matrix of the sampled observation sequence. What we would like to do is to test if the observations are compatible with the nominal model $A_0$. We define the following statistics

$$ H_k = RL_{k+1} A_0' $$

where $H_k A_0$ is the Hankel matrix defined by

$$ H_k = \begin{bmatrix} H_k & \ldots & H_{k-N} \end{bmatrix} \begin{bmatrix} \vdots \end{bmatrix} \begin{bmatrix} H_{k-N+1} \ldots H_{k-1} \end{bmatrix} $$

and $N$ is the number of instruments. The problem of detecting a change, in new equivalent to detecting a change in the AR model $A_0$. If we follow a local approach, that is if we suppose that the possible changes are of the form $\Theta = \Delta \Theta$ where $\Theta$ is a "direction" of change of $A_0$, then as the number $T$ of observations goes to infinity we can show the following central limit theorem [3].

**Theorem.** If $U_N$ is defined as in (8) then if $H_k \sim 0.1$ denotes the no change and the change hypothesis we have

$$ H_k < \sqrt{T/\Phi N(0, \Sigma_0)} $$

$$ H_k > \sqrt{T/\Phi N^2(\Sigma_{0.0} \cdot \Theta + N(0, \Sigma))} $$

where the convergence in (10) is in distribution. Thus under $H_0$ we have that $U_N$ behaves as a centered Gaussian matrix and under $H_1$ as non centered. The covariance matrix $\Sigma_0$ can be estimated using the covariances $R_k$ for details see [4,5]. Clearly now in order to test between the two hypotheses we use (10) as equality and thus the detection problem reduces to detecting a change in the mean of a Gaussian random vector. This problem is a classical detection problem. If we do not know anything about the direction $\Theta$ then we maximize the likelihood with respect to $\Theta$ and this yields a $\Phi^2$ statistic which for the case of $N = m$ (number of instruments equal to the AR order) takes the form

$$ T = w^T E_0 w $$

where $w$ is the vector which we get if we put the columns of $E_0$ one under another. A change is signaled if $T$ exceeds a threshold. The test defined this way is used as "global" since the only information that it gives is
whether there is a change or not. If we like now to test whether the mean has changed according to some pre-specified direction $\Theta$ with an unknown magnitude $\delta$ then we maximize the likelihood only with respect to the scalar $\delta$. This yields the following test statistic

$$T_\Theta = \frac{(T_\Theta^T E_1 X_{r-1, k} \cdot \Theta)^2}{\Theta^T H_{r-1, k} \cdot \Theta + \delta^2 H_{r-1, k} \cdot \delta}$$ (12)

where again $\Theta$ is the vector version of $\Theta$. Comparing $T_\Theta$ and $T_\delta$ we can see that

$$T_\Theta > T_\delta$$ (13)

The direction $\Theta$ does not necessarily correspond to the actual direction of change. In any case, we expect that if the change was indeed in the direction of $\Theta$ or to one "very close" to $\Theta$, to have a $T_\Theta$ very close to the global test statistic $T_\delta$. On the other hand if $\Theta$ is "far" from the direction of the actual change to have a $T_\Theta$ much smaller than $T_\delta$. This idea can be used to test the change of any other parameter related to our system. Assuming that the change of this parameter is small it will produce a variation in the AR model $A$ which will be equal to some matrix $\Theta$ times an unknown magnitude. This matrix $\Theta$ is the derivative of the AR model with respect to the parameter under consideration (first order approximation of $A$ around $A_0$). This diagonal method is called "sensitivity method" for the obvious reason and was applied in [4.5] to test which modes have been changed, once an alarm is signaled. We will use the same idea for detecting changes in the $K, K$ matrices.

IV. THE SENSITIVITY METHOD FOR THE $K, K$ MATRICES.

As we have seen in section III. all we can detect is changes in the AR model. So any change we like to test must be transformed into a corresponding change in the AR parameters. In other words if we like to test if we have a specific change in a mass or an elasticity we have to compute the derivative $\Theta$ of the AR model $A$ with respect to the corresponding mass or elasticity. Unfortunately such a computation needs not only the knowledge of the the excited eigenvalues and the observable part of their corresponding eigenvectors (fanges that can be computed directly from $A_0$ and thus from the data), but rather the knowledges of the whole eigenvectors. This is where the model $N, K, K$ and $A_0$ comes in. In other words we will not compute the derivative of $A_0$ but the derivative of $A$, defined by equation (7). Taking thus derivatives in (7) gives

$$(\delta A_0^\prime) = (\delta A)A_0 + \Theta (\delta A)$$ (14)

where $\delta X$ denotes the derivative of $X$ with respect to the quantity we want. In (14) $\delta A$ is exactly the direction we like to compute (what we call $\Theta$). From the definition of $\Theta$ we see that it is enough to compute $\delta A_0$. Since $\delta A_0 = E_1 X_{r-1, k}$ taking derivatives gives

$$\delta A_0 = E_1 (\delta X_{r-1, k})^T + \bar{\Theta} \cdot \delta X_{r-1, k}$$ (15)

From (15) we see that in order to compute the derivative of $\delta A_0$ we need the derivatives of the eigenvalues and the eigenvectors. In order to make this computation we use a method similar to the one used in [8].

Computation of changes of eigenvalues and eigenvectors.

Let $\lambda$ and $\phi$ be an eigenvalue and its corresponding eigenvector then we have that

$$(\delta \lambda^0 + \lambda \phi + \phi \delta \lambda) = 0$$ (16)

Taking derivatives in (16) gives
\[(e) \quad (DA + C) \psi + (\lambda^2 A^2 + k)x \psi + (\lambda^2 (DA) + (kC)) \psi = 0 \quad \text{(17)}\]

In (17) we have finally the quantities \(DA\) and \(DK\) we are interested in. Now multiplying (17) from the left by \(\psi^T\) and using (4) gives:

\[DK = \psi^T (DA^2 + k) \psi\]

Using the expression of \(DK\) from (18) in (17), gives a linear system in \(\psi\). The system has not a unique solution because \(\psi \psi^T\) is a solution so \(\psi \psi^T + \psi^T \psi\). We can thus look for the solution that is orthogonal to \(\psi\), i.e.

\[\psi^T \psi \psi = 0\]

We can now substitute for example one of these equations in (17) with (19) and obtain a system with a solution that is orthogonal to the eigenvector \(\psi\).

Actually condition (19) is equivalent to saying that \(\psi^T \psi \psi = 0\), i.e. the change in the eigenvector \(\psi + \psi\) that is as close as possible to the maximum sense to \(\psi\). In any case we can verify that by taking any solution of (17) does not change \(DK\) computed from (14).

Using Equations (17, 18, 19) we can find \(DK\) and \(DK\) and then from (14) \(DK\) for different terms of \(DA\) and \(DK\) matrices. The most common form for \(DK\) is \(\sigma_1 \sigma_2\) for a change in the \(1,1\) diagonal element \(\sigma_1\) is a vector with all its elements equal to zero except the \(1,1\) th which is equal to unity. For \(DK\) we have \((\sigma_i - \sigma_j)(\sigma_i - \sigma_j)^T\) for a change in the cross elasticity between points \(k\) and \(j\), or \(\sigma_i \sigma_j\) for the auto elasticity at the point \(i\).

Let us now assume that we have a collection of directions \(\theta_1, \ldots, \theta_k\) corresponding each one to some change in the matrices \(M, K\). Since in general the representation \(MK\) has much more parameters than the representation of \(A\) we expect that some of the directions \(\theta_i\) will fall close to each other and we cannot test statistic \(T_k\) defined by (12) we will not be able to distinguish between them. We thus need to classify our directions into clusters and for each cluster to use a representative. So defining clusters we need first to define a distance measure that has some meaning with respect to our test.

### Definition of a Distance Measure

As we said the measure we want to define must have a close relation with our diagnostic method. Let us thus assume that we have an actual change in the direction \(\theta_i\) and that we look in the direction \(\theta_j\) using the statistics defined by (12). First notice that in deriving the test we assumed that the change is of the form \(\theta_i \theta_j^{1/2}\). That is, the magnitude of the change goes to zero as the number of observations increases. This assumption is always made whenever we follow a local approach. In practice though it is unrealistic since a change has usually a zero magnitude. Assuming that this is the case, the random matrix \(U_{ij}\) will have a mean that will go to infinity as the number of observations increases, and thus the matrix will fluctuate around this non-random value that goes to infinity. Since this increase in \(U_{ij}\) is in the limit \(U_{ij}\) properly normalized by \(T\) will tend to 0. Mean thus the expression of the statistics (11) and (12) can be substituted \(U_{ij}\) with its mean. This gives:

\[T_k = \frac{\sigma_i^T \sigma_j^{-1/2}}{\sigma_j^{1/2} \sigma_i^{-1/2}} \theta_i \]

\[T_k = \frac{\sigma_i^T \sigma_j^{-1/2} \theta_i \theta_j^{1/2} \sigma_j^{-1/2} \theta_i}{\sigma_j^{1/2} \sigma_i^{-1/2} \theta_i \theta_i^{1/2} \sigma_i^{-1/2} \theta_i} \]

Considering in (20) the approximate equalities as equations and computing \(N\) and \(\Sigma\) using the observations under \(n_i\) we have again \(T_k / T\). We can define now the quantity
\[ \rho_{ij} = \frac{\left( \frac{\partial f}{\partial x_i} \right)^2 \left( \frac{\partial f}{\partial x_j} \right)^2}{\left( \frac{\partial f}{\partial x_i} \right)^2 + \left( \frac{\partial f}{\partial x_j} \right)^2} \]  

(21)

as a proximity measure between \( \Theta_i \) and \( \Theta_j \). A distance measure can be easily defined from \( \rho_{ij} \). For example we can use \( \delta_{ij} = 1 - \rho_{ij} \) using this definition of distance we can use classical clustering algorithms and classify our directions. Then to each class we correspond a representation (the mean direction for example) and use these representatives to make a diagnosis. Clearly by selecting a specific representative as the cause of the alarm we select a whole class of possible changes, which is being able to distinguish among them. For certain applications this is enough as information.

V. CONCLUSION.

We have presented here a method of diagnosis with respect to the physical parameters of a structure when an alarm for a change is signaled. This method is very closely related to the Instrumental Variable Detection Method and actually directs the test statistics to certain precomputed subspaces. The relation between the subspaces and the physical parameters is known a priori, so the selection of a specific subspace corresponds to a selection of a set of possible physical parameters. As it is seen in Section IV, we are interested only in changes with respect to the mass and elasticity matrix elements. This is not restrictive since for large structures the damping matrix is very close to zero and considering changes with respect to this matrix is useless.

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