Due by the beginning of class, Dec. 2.

1. When a key is inserted into a skip list, it may be inserted into several lists – as many as \( O(\log n) \). The size of a skip list is the sum of the lengths of all of its lists. Give tight bounds for the size of an \( n \) node skip list.

2. Keys with large height (that is, they are in many lists) in a skip list are faster to find than average keys. Suppose that you are have implemented a skip list and Eve has a stop watch.

Alice is running the skip list and Bob is using the skip list (that is, Bob is issuing commands like add, delete, succ, pred). Even is sitting on the communication channel and can measure Alice’s reaction times and can replace Bob’s commands with other commands. Her only restriction is that when Bob issues a successor command or a predecessor command, he has to get the right answer.

How badly can Even screw up the skip list? What can Alice to thwart Eve? How much overhead does Alice’s evasive maneuvers add?

3. The Metric Sandwich Problem is defined as follows. Given two \( n \times n \) matrices \( M_l \) and \( M_h \), is there a metric that is dominated by \( M_h \) and dominates \( M_l \)? That is, is there a metric that is sandwiched between \( M_h \) and \( M_l \)? Give an algorithm to solve this problem. The faster the algorithm, the better.