Due by the beginning of class, Oct. 27.

1. Let $T \in \Sigma^n$ and $P \in \Sigma^m$. For each position $i$, you’d like to find the Hamming distance between $P$ and $T[i \ldots i + m - 1]$. Give an algorithm for finding all such Hamming distances. You might try an FFT approach.

2. Suppose that you have an array $A[1, \ldots, n]$ of numbers.
   
   (a) Show how to randomly permute $A$ so that every permutation occurs with equal probability. The faster the algorithm, the better.
   
   (b) Suppose that you randomly permute $A$ and then insert the elements of $A$ in this random order into a search tree. Suppose this is not a self-adjusting tree but that each key is inserted as a leaf and ends up wherever it ends up from this insertion. Show that the expected depth of the tree is $O(\log n)$. Better yet, show that the depth is $O(\log n)$ with high probability.

3. Define a metric to be a function $F : S \times S \to \mathbb{R}$ such that $F(x, y) = 0$ iff $x = y$, $F(x, y) = F(y, x)$ and $F(x, z) \leq F(x, y) + F(y, z)$.

   Let $G$ be any weighted undirected graph. Let $S_G$ be the all-pairs-shortest paths of $G$, that is $S_G(u, v) =$ the length of the shortest path from $u$ to $v$ in $G$.

   (a) Assume that $G$ is connected. Show that $S_G$ is a metric.
   
   (b) Assume further that $G$ is a complete graph. Show that if $F$ is a metric on the nodes of $F$, and that $G$ dominates $F$, which means that for any $u$ and $v$, $G(u, v) \geq F(u, v)$, where $G(u, v)$ is the weight of the edge from $u$ to $v$ in $G$. Show that $S_G$ dominates $F$. 