Outlines

- Point Operations
- Brightness and contrast adjustment
- Auto contrast
- Histogram equalization
- Histogram specification

- Source: Burger & Burge “Digital Image Processing”
Point Operations

- Point Operations perform a mapping of the pixel values without changing the size, geometry, or local structure of the image.
- Each new pixel value $I'(u,v)$ depends on the previous value $I(u,v)$ at the same position and on a mapping function $f()$.
- The function $f()$ is independent of the coordinates.
- Such operation is called “homogeneous”.

\[
a' \leftarrow f(a) \\
I'(u,v) \leftarrow f(I(u,v))
\]

Example of homogeneous point operations:

- Modifying image brightness or contrast
- Applying arbitrary intensity transformation (curves)
- Quantizing (posterizing) images
- Global thresholding
- Gamma correction
- Color transformations
A nonhomogeneous point operation $g()$ would also take into account the current image coordinate $(u,v)$

\[
\begin{align*}
\alpha' & \leftarrow g(\alpha, u, v) \\
I'(u, v) & \leftarrow g(I(u, v), u, v)
\end{align*}
\]

- Changing contrast and brightness
  \[f_{\text{contr}}(a) = a \cdot 1.5 \quad \text{and} \quad f_{\text{bright}}(a) = a + 10\]
- Limiting Results by Clamping

```java
public void run(ImageProcessor ip) {
    int w = ip.width();
    int h = ip.height();

    for (int v = 0; v < h; v++) {
        for (int u = 0; u < w; u++) {
            int a = (int) (ip.get(u, v) + 1.5 + 0.5);
            if (a > 255)
                a = 255;  // clamp to maximum value
            ip.set(u, v, a);
        }
    }
}
```
Threshold Operation

- Thresholding an image is a special type of quantization that separates the pixel values in two classes, depending on a given threshold value $a_{th}$.
- The threshold function maps all the pixels to one of two fixed intensity values $a_0, a_1$

$$f_{\text{threshold}}(a) = \begin{cases} 
  a_0 & \text{for } a < a_{th} \\
  a_1 & \text{for } a \geq a_{th} 
\end{cases}$$

$$0 < a_{th} \leq a_{\max}$$
- Example: binarization: $a_0 = 0, a_1 = 1$
Point Operations and Histograms

- The effect of some point operations on histograms are easy to predict: ex: increasing the brightness, raising the contrast, inverting an image
- Point operations can only shift and merge histogram entries
- Operations that result in merging histogram bins are irreversible
Automatic Contrast Adjustment

- Auto-contrast: a point operation that modifies the pixels such that the available range of values is fully covered.
- Linear stretching of the intensity range - can result in gaps in the new histogram

\[
\begin{align*}
\tilde{f}(a) &= a_{\text{low}} + (a - a_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{a_{\text{high}} - a_{\text{low}}} \\
\tilde{f}(a) &= (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}
\end{align*}
\]

![Diagram of contrast adjustment](image)
Better Auto-contrast

- It’s better to map only a certain range of the values and get rid of the tails (usually noise) based on predefined percentiles ($s_{\text{low}}$, $s_{\text{high}}$)

$$\hat{a}_{\text{low}} = \min \{ i \mid H(i) \geq M \cdot N \cdot s_{\text{low}} \}$$

$$\hat{a}_{\text{high}} = \max \{ i \mid H(i) \leq M \cdot N \cdot (1 - s_{\text{high}}) \}$$

$$f_{\text{max}}(a) = \begin{cases} a_{\text{min}} & \text{for } a \leq \hat{a}_{\text{low}} \\ a_{\text{min}} + (a - \hat{a}_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{\hat{a}_{\text{high}} - \hat{a}_{\text{low}}} & \text{for } \hat{a}_{\text{low}} < a < \hat{a}_{\text{high}} \\ a_{\text{max}} & \text{for } a \geq \hat{a}_{\text{high}} \end{cases}$$

Histogram Equalization

- Adjust two different images in such a way that their resulting intensity distribution are similar
- Useful when comparing images to get rid of illumination variations
- The goal is to find and apply a point operation such that the histogram of the modified image approximates a uniform distribution.
- Linear Histogram equalization

\[ f_{eq}(a) = \frac{H(a) \cdot K - 1}{MN} \]
Histogram Specification

- Real images never show uniform distribution
- In most real images the distribution of pixel values is more similar to a Gaussian Distribution
- Histogram specification modifies the image to match an arbitrary intensity distribution, including the histogram of a given image.
- Also depends on the alignment of the cumulative histograms by applying a homogeneous point operation.

Histogram Specification

- Find a mapping such that
  \[ a' = f_{bs}(a) \]

\[ P_{A'}(i) \approx P_R(i) \quad \text{for} \quad 0 \leq i < K \]

\[ f_{bs}(a) = a' = P_R^{-1}(P_A(a)) \]
Adjusting piecewise linear distribution

\[ \mathcal{L} = [(a_0, q_0), (a_1, q_1), \ldots, (a_K, q_K), \ldots, (a_N, q_N)] \]

\[ \langle 0, q_0 \rangle \quad \text{and} \quad \langle K-1, 1 \rangle \]

\[ P_L(i) = \begin{cases} 
q_m + (i-a_m) \frac{(q_{m+1} - q_m)}{(a_{m+1} - a_m)} & \text{for } 0 \leq i < K-1 \\
1 & \text{for } i = K-1
\end{cases} \]

\[ P_L^{-1}(b) = \begin{cases} 
0 & \text{for } 0 \leq b < P_L(0) \\
a_m + (b-q_m) \frac{(a_{m+1} - a_m)}{(q_{m+1} - q_m)} & \text{for } P_L(0) \leq b < 1 \\
K-1 & \text{for } b \geq 1
\end{cases} \]

\[ n = \max\{j \in \{0, \ldots, N-1\} \mid q_j \leq b\}. \]
1: Piecewise Linear Interpolant(h, K, K̃)

$h_1$: histogram of the original image.

$K̃$: reference distribution function, given as a sequence of $K + 1$
colour intervals $K̃ = \{(a_0, q_0), (a_1, q_1), \ldots, (a_K, q_K)\}$, with $0 \leq a_k < K$
and $0 \leq q_k \leq 1$.

2: Let $K = \text{Size}(h)$.
3: Let $P_a = \text{Core}(h)$.
4: Create a table $f_{al}$ of size $K$.
5: for $a = P_a[1]$ do
6: $\delta \leftarrow P_a[2]$
7: if $(a \leq q_a)$ then
8: $a' \leftarrow a$
9: else if $(a > q_a)$ then
10: $a' \leftarrow K - 1$
11: end
12: $\psi \leftarrow N - 1$
13: while $(a \geq \psi) \land (a > a')$ do
14: $\psi \leftarrow \psi - 1$
15: $a' \leftarrow a + (b - a) \cdot \frac{(q_{\psi + 1} - a_\psi)}{(q_a - a_\psi)}$
16: end
17: return $f_{al}$.

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Original Image

Modified Image

Reference Distribution (piecewise linear).

(a) $I_a$

(b) $I_a'$

(c) $h_1$

(d) $h_1$

(e) $h_1$

(f) $P_a$

(g) $P_a$

(h) $P_a'$
Adjusting to a given histogram

\[ f_{\text{hl}}(a) = d' = \min \{ j \mid 0 \leq j < K \land (P_A(a) \leq P_R(j)) \} \]

```
1: MATCHHISTOGRAM(h_A, h_R)  
   h_A: histogram of the target image  
   h_R: reference histogram (of course also on h_A)  
2:  Let K ← size(h_A)  
3:  Let P_A ← Uniform(h_A)  
4:  Let P_R ← Uniform(h_R)  
5:  Create a table \( f_{\text{hl}} \) of size \( K \)  
   \( f_{\text{hl}} \) pixel mapping function \( f_{\text{hl}} \)  
6:  for \( i = 0 \ldots (K-1) \) do  
7:     \( j ← K-1 \)  
8:     repeat  
9:     \( f_{\text{hl}}[i] ← j \)  
10:    \( j ← j - 1 \)  
11:  while \( (j \geq 0) \land (P_A(a) \leq P_R(j)) \)  
12:  return \( f_{\text{hl}} \).
```
Gamma Correction

- What is the relation between the amount of light falling onto a sensor and the “intensity” or “brightness” measured at the corresponding pixel.
- What is the relation between the intensity of a pixel and the actual light emanating from that pixel on the display?
- The relation between a pixel value and the corresponding physical quantity is usually complex and nonlinear.
- Approximation?

What is Gamma?

- Originates from analog photography
- Exposure function: the relationship between the logarithmic light intensity and the resulting film density.
- Gamma is the slope of the linear range of the curve.
- The same in TV broadcasting
The Gamma function

- Gamma function is a good approximation for the exposure curve.
- The inverse of a Gamma function is another gamma function with
  \[ \tilde{\gamma} = 1/\gamma \]
- Gamma of CRT and LCD monitors: 1.8-2.8 (typically 2.4)

\[ b = f_{\gamma}(a) = a^{\gamma} \text{ for } a \in \mathbb{R}, \gamma > 0 \]

\[ a = f_{\gamma}^{-1}(b) = b^{1/\gamma} \]

\[ f_{\gamma}^{-1}(b) = f_{\gamma}(b) \]

\[ \tilde{\gamma} = 1/\gamma \]

Gamma Correction

- Obtain a measurement \( b \) proportional to the original light intensity \( B \) by applying the inverse gamma function
- This is important to achieve a device independent representation

\[ B \rightarrow \hat{B} \rightarrow f_{\gamma}(s, \hat{\gamma}) \rightarrow \tilde{b} \approx B \]
Gamma Correction

\[ \gamma_c = 1.3 \]
\[ \gamma_s = \frac{1}{1.9} \]
\[ \gamma_p = 3.0 \]
\[ \gamma_m = 2.6 \]