Final Examination
CS 452/508: Formal Languages and Automata Theory
Spring Semester, 2021 This is a take-home final.
It is due at noon on Thursday, May 6
LATE PAPERS WILL NOT BE ACCEPTED.

INSTRUCTIONS:

• Write the following sentence on the top of your exam. On my honor, I have neither received nor given any unauthorized assistance on this examination.

• Work individually. You may consult the textbook, but you should not consult other texts, or the internet, or other experts, or each other. If you have questions or want to discuss some of the problems, then contact me. I will be glad to discuss things with you. Answers that seem too similar to each other or to material that is posted on the internet (for example) may cause your exam to receive a failing grade; don’t risk it. You are encouraged to contact me to discuss the exam questions.

• Submit your completed exam by noon on Thursday, May 6 (in the same way that you submit your homework assignments).

• Show your work and provide some explanations; you get little or no credit for an unexplained answer.

• The value of each question appears in parentheses. There are 150 points in total.

1. (1 point) Write “On my honor, I have neither received nor given any unauthorized assistance on this examination.”

2. (30 points) The problem ALL_{NFA} is discussed in Example 8.4. The text states that this problem is not known to be in NP or in coNP. Show that this set is complete for one of the main complexity classes that we have discussed (L, NL, P, NP, PSPACE, EXPTIME, EXPSPACE) under logspace reductions.

3. (30 points) The problem ALL_{DFA} is just like ALL_{NFA}, except that the input is the encoding of a DFA. Show that this problem is complete for one of the main complexity classes that we have discussed (L, NL, P, NP, PSPACE, EXPTIME, EXPSPACE) under logspace reductions.

4. (29 points) For each of the following statements, say whether it is true or false, and explain your answer. (Hint: Your answers to problems 1 and 2 should determine your answers to problem 3. This is not related to some
open question in complexity theory; each part of problem 3 has a definite “true” or “false” answer.)

- \( \text{ALL}_{\text{NFA}} \leq_{L} \text{ALL}_{\text{DFA}} \).
- \( \text{ALL}_{\text{DFA}} \leq_{L} \text{ALL}_{\text{NFA}} \).

5. (30 points) We presented a polynomial-time algorithm that takes as input a description of a DFA and produces a minimal equivalent DFA. It is not known whether there is a polynomial-time algorithm that takes as input a description of an NFA and produces a minimal equivalent NFA.

- What can one conclude if there is a polynomial-time algorithm to minimize NFAs? (There was a homework assignment related to this. Can you derive an even stronger conclusion than was in the homework?)
- Is there a logspace-computable algorithm to minimize NFAs? (Why or why not?)
- Is there a polynomial-time algorithm that takes as input a description of an NFA and produces a minimal equivalent DFA? (Why or why not?)

6. (30 points) Show that, for every decidable set \( A \) there is a decidable set \( B \) such that \( B \) is not polynomial-time reducible to \( A \). Then show that this implies that there is no set that is complete for the class of Turing-decidable sets under \( \leq_{P} \) reductions. (Hint: You can use the fact that, for any computable function \( t \), there is a time-constructible function \( T \) such that \( T(n) > t(2^{n}) \).)