CS206 **Homework # 5** **Due October 20, 2020**

The first seven problems deal with the following "experiment". Assume that two teams x and y are going to play each other in the World Series of Baseball, and furthermore assume that team x has probability p of defeating team y in any given game, and that each game constitutes an independent trial. (The World Series ends after one team has won four games; there are no tie games in baseball.)

1. What is the probability that team x will win the first four games?
2. What is the probability that team x will win four games after at most five games have been played?
3. What is the probability that team x will win four games before team y (and thus win the World Series)?
4. Evaluate your answer to problem 3, when $p = 1/2$ and when $p = 2/3$.
5. Let X be the random variable that counts the total number of games in the world series. What is Range(X)?
6. What is $\Pr[X = 7]$?
7. What is $\Pr[X \geq 6]$?
8. A fair die is tossed twice. Let X = the sum of the faces, Y = the maximum of the two faces, and Z = the absolute value of the difference of the two rolls. Let $W = XZ$.
   (a) What are Range(X), Range(Y), Range(Z), and Range(W)?
   (b) What are the partitions of the sample space induced by the random variables X and Z?
   (c) Are the events $X = 7$ and $Z = 1$ independent?
9. Consider the "hat" experiment with three hats. Consider the event A = "neither man 1 nor man 2 gets their own hat". Let X be the indicator function for this event, and let Y count the number of men who do *not* get their own hat.
   (a) Find Range(X) and Range(Y).
   (b) There are 3! elements of the sample space. Give the partition induced by X and the partition induced by Y.
   (c) Are X and Y independent?
10. Experiment $\mathcal{E}$ is to toss a fair coin twice; the experiment succeeds if both tosses are Tails. The experiment is repeated for 12 independent trials. Let N count the fraction of the 12 trials that are successes. (That is, $N = S/12$ where S counts the number of successes.) Find $E(N)$.

Extra credit Three players A, B and C take turns tossing a pair of coins (with A going first, then B, then C, then A, and so on). The game ends when one of the players tosses two Heads in one turn. For each player, find the probability of that player winning.