Abstract

We study the problem of allocating a set of indivisible items among agents with additive valuations with the goal of maximizing the geometric mean of the agents valuations, i.e., the Nash social welfare. This problem is known to be NP-hard, and our main result is the first efficient constant-factor approximation algorithm for this objective. We first observe that the integrality gap of the natural fractional relaxation is unbounded, so we propose a different fractional allocation which implies a tighter upper bound and, after appropriate rounding, yields a good integral allocation.

An interesting contribution of this work is the fractional allocation that we use. The relaxation of our problem can be solved efficiently using the Eisenberg-Gale program, whose optimal solution can be interpreted as a market equilibrium with the dual variables playing the role of item prices. Using this market-based interpretation, we define an alternative equilibrium allocation where the amount of spending that can go into any given item is bounded, thus keeping the highly priced items under-allocated, and forcing the agents to spend on lower priced items. The resulting equilibrium allocation reveals more information regarding how to assign items so as to obtain a good integral allocation.

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