Class Announcements

- Hope you had a good Spring Break!
- Fourth homework: Deadline extension - Now Wednesday, March 29. Parameter passing styles will be discussed in recitation
- First project: Deadline extension - Now Monday, March 27.
- Functional programming resources: Online book on Scheme; we will use racket and dracket which you can install on your laptop or machine. Both interpreters are available on our ilab cluster.
[Functional Programming]

Pure Functional Languages

Scott: Chapter 10, 11

Fundamental concept: application of (mathematical) functions to values

1. Referential transparency: The value of a function application is independent of the context in which it occurs
   - value of $f(a, b, c)$ depends only on the values of $f, a, b$ and $c$
   - It does not depend on the global state of computation
   $\Rightarrow$ all vars in function must be local (or parameters)
2. The concept of assignment is **not** part of functional programming
   - no explicit assignment statements
   - variables bound to values only through the association of actual parameters to formal parameters in function calls
   - function calls have no side effects
   - thus no need to consider global state

3. Control flow is governed by function calls and conditional expressions
   ⇒ no iteration
   ⇒ recursion is widely used
Pure Functional Languages

4. All storage management is implicit
   - needs garbage collection

5. Functions are *First Class Values*
   - Can be returned as the value of an expression
   - Can be passed as an argument
   - Can be put in a data structure as a value
   - (Unnamed) functions exist as values
Pure Functional Languages

A program includes:

1. A set of function definitions
2. An expression to be evaluated

E.g. in Scheme:

```
> (define length
   (lambda (x)
     (if (null? x)
       0
       (+ 1 (length (rest x))))))

> (length '(A LIST OF 5 THINGS))
5
```

We will use Scheme (racket - (command line interpreter) or drracket (Integrated Development Environment)) for the second project.
LISP

- Functional language developed by John McCarthy in the mid 50’s
- Semantics based on Lambda Calculus
- All functions operate on lists or symbols: (called “S-expressions”)
- Only five basic functions: list functions cons, car, cdr, equal, atom and one conditional construct: cond
- Useful for list-processing applications
- Programs and data have the same syntactic form: S-expressions
- Used in Artificial Intelligence
Lambda calculus

- formalism for studying ways in which functions can be formed, combined, and used for computation

- **computation** is defined as rewriting rules (operational semantics) $\Rightarrow \beta$ reduction

- the syntactic notion of computation was developed first; a mathematical semantics followed much later

Examples:

- $f(x) = x+2$  \hspace{1cm} $(\lambda x.x+2)$  different notation
- $((\lambda x.x+2) 1)$  \hspace{1cm} $1+2 = 3$  function application and substitution
- $((\lambda x.x) (\lambda y.y))$  \hspace{1cm} arguments and returned “values” can be functions
- $(\lambda x.xx)$  \hspace{1cm} untyped lambda calculus
  \hspace{1cm} $f(x) = x(x)$
SCHEME

- Developed in 1975 by G. Sussman and G. Steele
- A version of LISP
- Simple syntax, small language
- Closer to initial semantics of LISP as compared to COMMON LISP
- Provides basic list processing tools
- Allows functions to be first class objects
SCHEME

- Expressions are written in prefix, parenthesized form
  
  - (function arg₁ arg₂ ...argₙ)
  
  - (+ 4 5)
  
  - (+ (* 3 4 5) (- 5 3))

- Operational semantics: In order to evaluate an expression:
  
  1. evaluate function to a function value
  2. evaluate each argᵢ in order to obtain its value
  3. apply the function value to these values
S-expressions

S-expression ::= Atom | `( S-expression `)
Atom ::= Name | Number | #t | #f

#t

()

(a b c)

(a (b c) d)

((a b c) (d e (f)))

(1 (b) 2)

Lists have nested structure.
Lists in Scheme

The building blocks for lists are **pairs** or **cons-cells**. Lists use the empty list ( ) as an “end-of-list” marker.

Note: \((a.b)\) is not a list!
Special (Primitive) Functions

- eq?: identity on names (atoms)
- null?: is list empty?
- car: selects first element of list (contents of address part of register)
- cdr: selects rest of list (contents of decrement part of register)
- (cons element list): constructs lists by adding element to front of list
- quote or ’: produces constants
Special (Primitive) Functions

- '() is the empty list
- (car '(a b c)) =
- (car '((a) b (c d))) =
- (cdr '(a b c)) =
- (cdr '((a) b (c d))) =
• **car** and **cdr** can break up any list:

- \( (\text{car} \ (\text{cdr} \ (\text{cdr} \ '((a) \ b \ (c \ d)))))) = \)

- \( (\text{caddr} \ '((a) \ b \ (c \ d))) \)

• **cons** can construct any list:

- \( (\text{cons} \ 'a \ '()) = \)

- \( (\text{cons} \ 'd \ '(e)) = \)

- \( (\text{cons} \ '((a) \ b) \ '((c) \ d)) = \)

- \( (\text{cons} \ '((a) \ b \ c) \ '((a) \ b)) = \)
Other Functions

- + - * / numeric operators, e.g.,
  \[(+ 5 3) = 8, (- 5 3) = 2\]
  \[(* 5 3) = 15, (/ 5 3) = 1.6666666\]
- = < > comparison operators for numbers

- Explicit type determination and test functions:
  \[\Rightarrow \text{All return Boolean values: } \#f \text{ and } \#t\]
  - (number? 5) evaluates to \#t
  - (zero? 0) evaluates to \#t
  - (symbol? 'sam) evaluates to \#t
  - (list? '(a b)) evaluates to \#t
  - (null? '()) evaluates to \#t

Note: SCHEME is a strongly typed language.
Other Functions

- `(number? 'sam)` evaluates to `#f`
- `(null? '(a))` evaluates to `#f`
- `(zero? (- 3 3))` evaluates to `#t`
- `(zero? '(- 3 3))` ⇒ type error
- `(list? (+ 3 4))` evaluates to `#f`
- `(list? '(+ 3 4))` evaluates to `#t`
READ-EVAL-PRINT Loop

The Scheme interpreters on the ilab machines are called \texttt{mzscheme}, \texttt{racket}, and \texttt{drracket}. “drracket” is an interactive environment, the others are command-line based. For example: Type \texttt{mzscheme}, and you are in the READ-EVAL-PRINT loop. Use \texttt{Control D} to exit the interpreter.

\textbf{READ}: Read input from user:

\begin{itemize}
  \item a function application
\end{itemize}

\textbf{EVAL}: Evaluate input:

\[(f \ \text{arg}_1 \ \text{arg}_2 \ \ldots \ \text{arg}_n)\]

\begin{enumerate}
  \item evaluate \texttt{f} to obtain a function
  \item evaluate each \texttt{arg}_i to obtain a value
  \item apply function to argument values
\end{enumerate}

\textbf{PRINT}: Print resulting value:

\begin{itemize}
  \item the result of the function application
\end{itemize}

You can write your Scheme program in file \texttt{<name>.ss} and then read it into the Scheme interpreter by saying at the interpreter prompt: \texttt{(load "<name>.ss")}
> (cons 'a (cons 'b '(c d)))
(a b c d)

1. Read the function application
   (cons 'a (cons 'b '(c d)))

2. Evaluate cons to obtain a function

3. Evaluate 'a to obtain a itself

4. Evaluate (cons 'b '(c d)):
   (a) Evaluate cons to obtain a function
   (b) Evaluate 'b to obtain b itself
   (c) Evaluate '(c d) to obtain (c d) itself
   (d) Apply the cons function to b and (c d) to obtain (b c d)

5. Apply the cons function to a and (b c d) to obtain (a b c d)

6. Print the result of the application:
   (a b c d)
Quotes Inhibit Evaluation

;; Same as before:
> (cons 'a (cons 'b '(c d)))
(a b c d)

;; Now quote the second argument:
> (cons 'a '(cons 'b '(c d)))
(a cons (quote b) (quote (c d)))

;; Instead, un-quote the first argument:
> (cons a (cons 'b '(c d)))
ERROR: unbound variable: a
Defining Global Variables

The define constructs extends the current interpreter environment by the new defined (name, value) association.

> (define foo '(a b c))
#<unspecified>

> (define bar '(d e f))
#<unspecified>

> (append foo bar)
(a b c d e f)

> (cons foo bar)
((a b c) d e f)

> (cons 'foo bar)
(foo d e f)
(define <fcn-name> (lambda (<fcn-params>)
  <expression>))

Example: Given function pair? (true for non-empty lists, false o/w) and function not (boolean negation):

(define atom?
  (lambda (object) (not (pair? object))))

Evaluating (atom? '(a)):
1. Obtain function value for atom?
2. Evaluate '(a) obtaining (a)
3. Evaluate (not (pair? object))
   a) Obtain function value for not
   b) Evaluate (pair? object)
      i. Obtain function value for pair?
      ii. Evaluate object obtaining (a)
         Evaluates to #t
         Evaluates to #f
Evaluates to #f
Conditional Execution: if

(if <condition> <result1> <result2>)

1. Evaluate <condition>

2. If the result is a “true value” (i.e., anything but #f), then evaluate and return <result1>

3. Otherwise, evaluate and return <result2>

(define abs-val
  (lambda (x)
    (if (>= x 0) x (- x)))))
Conditional Execution: cond

(cond (<condition1> <result1>)
  (<condition2> <result2>)
  ...
  (<conditionN> <resultN>))
(else <else-result>)); optional else ; clause

1. Evaluate conditions in order until obtaining one that returns a true value
2. Evaluate and return the corresponding result
3. If none of the conditions returns a true value, evaluate and return <else-result>
Recursive Scheme Functions: Abs-List

- (abs-list '(1 -2 -3 4 0)) ⇒ (1 2 3 4 0)
- (abs-list '()) ⇒ ()

(define abs-list
  (lambda (l)
    (if (null? l)
      '()
      (cons (abs-val (car l)) (abs-list (cdr l))))))
Recursive Scheme Functions: Append

(append '(1 2) '(3 4 5) ⇒ (1 2 3 4 5)
(append '(1 2) '(3 (4) 5) ⇒ (1 2 3 (4) 5)
(append () '(1 4 5)) ⇒ (1 4 5)
(append '(1 4 5) '()) ⇒ (1 4 5)
(append '() '()) ⇒ ()

(define append
  (lambda (x y) )
)
Next time:

- More programming in Scheme
- Lambda calculus
Equality Checking

The `eq?` predicate doesn’t work for lists.
Why not?

1. `(cons 'a '())` produces a new list
2. `(cons 'a '())` produces another new list
3. `eq?` checks if its two arguments are the same
4. `(eq? (cons 'a '()) (cons 'a '()))` evaluates to `#f`

Lists are stored as pointers to the first element (car) and the rest of the list (cdr). This elementary “data structure”, the building block of lists, is called a pair.

Symbols are stored uniquely, so `eq?` works on them.
Equality Checking for Lists

For lists, need a comparison function to check for the same structure in two lists

```scheme
(define equal?
  (lambda (x y)
    (or (and (atom? x) (atom? y) (eq? x y))
      (and (not (atom? x)) (not (atom? y))
       (equal? (car x) (car y))
       (equal? (cdr x) (cdr y))))))
```

- `(equal? 'a 'a)` evaluates to `#t`
- `(equal? 'a 'b)` evaluates to `#f`
- `(equal? '(a) '(a))` evaluates to `#t`
- `(equal? '((a)) '(a))` evaluates to `#f`
Next Lecture

Next time:

- More programming in Scheme
- Lambda calculus