

CS 314 Principles of Programming Languages

Lecture 8: LL(1) Parsing

Zheng (Eddy) Zhang



Rutgers University

February 12, 2018

Class Information

- Homework 3 posted, due Sunday 2/18 11:55 pm EST.

Review: LL(1) Predictive Parsing

Key Property:

Whenever two productions $A ::= \alpha$ and $A ::= \beta$ both appear in the grammar, we would like

- $FIRST(\alpha) \cap FIRST(\beta) = \emptyset$, and
- if $\alpha \Rightarrow^* \varepsilon$, then $FIRST(\beta) \cap FOLLOW(A) = \emptyset$

Analogue case for $\beta \Rightarrow^* \varepsilon$.

Note: due to first condition, at most one of α and β can derive ε .

This would allow the parser to make a correct choice with a lookahead of only one symbol!

FIRST and FOLLOW Sets

FIRST(α):

For some $\alpha \in (T \cup NT \cup EOF \cup \varepsilon)^*$, define **FIRST** (α) as the set of tokens that appear as the first symbol in some string that derives from α .

That is, $x \in \text{FIRST}(\alpha)$ iff $\alpha \Rightarrow^* x\gamma$ for some γ

FIRST set is defined over the strings of grammar symbols
 $(T \cup NT \cup EOF \cup \varepsilon)^*$

T: terminals NT: non-terminals

First Set Example

Start ::= S eof

S ::= a S b | ϵ

$FIRST(aSb) = \{a\}$

$FIRST(\epsilon) = \{\epsilon\}$

$FIRST(S) = \{a, \epsilon\}$

FIRST and FOLLOW Sets

FOLLOW(A):

For $A \in \mathbf{NT}$, define **FOLLOW(A)** as the set of *tokens* that can occur immediately after A in a valid sentential form.

FOLLOW set is defined over the set of non-terminal symbols, **NT**.

Back to Our Example

Start ::= S eof

S ::= a S b |
ε

$FOLLOW(S) = \{ \text{eof}, b \}$

First Set Construction

Build $\text{FIRST}(X)$ for all grammar symbols X :

- For each X as a terminal, then $\text{FIRST}(X)$ is $\{X\}$
- If $X ::= \varepsilon$, then $\varepsilon \in \text{FIRST}(X)$
- For each X as a non-terminal, initialize $\text{FIRST}(X)$ to \emptyset
- ***Iterate until*** no more terminals or ε can be added to any $\text{FIRST}(X)$:

For each rule in the grammar of the form $X ::= Y_1 Y_2 \dots Y_k$

add a to $\text{FIRST}(X)$ if $a \in \text{FIRST}(Y_1)$

add a to $\text{FIRST}(X)$ if $a \in \text{FIRST}(Y_i)$ and $\varepsilon \in \text{FIRST}(Y_j)$

for all $1 \leq j \leq i-1$ and $i \geq 2$

add ε to $\text{FIRST}(X)$ if $\varepsilon \in \text{FIRST}(Y_i)$ for all $1 \leq i \leq k$

EndFor

End iterate

Filling in the Details: Computing *FIRST* sets

for each $x \in (T \cup \text{EOF} \cup \varepsilon)$

$\mathbf{FIRST}(x) \leftarrow \{x\}$

for each $A \in \mathbf{NT}$, $\mathbf{FIRST}(A) \leftarrow \emptyset$

Initially, set *FIRST* for each terminal symbol, EOF and ε

while (*FIRST* sets are still changing) do

for each $p \in P$, of the form $X \rightarrow Y_1 Y_2 \dots Y_k$ do

temp $\leftarrow \mathbf{FIRST}(Y_1) - \{ \varepsilon \}$

$i \leftarrow 1$

while ($i \leq k-1$ and $\varepsilon \in \mathbf{FIRST}(Y_i)$)

temp \leftarrow temp \cup ($\mathbf{FIRST}(Y_{i+1}) - \{ \varepsilon \}$)

$i \leftarrow i + 1$

end // while loop

if $i == k$ and $\varepsilon \in \mathbf{FIRST}(Y_k)$

then temp \leftarrow temp \cup $\{ \varepsilon \}$

$\mathbf{FIRST}(X) \leftarrow \mathbf{FIRST}(X) \cup$ temp

end // if - then

end // for loop

end // while loop

Filling in the Details: Computing *FIRST* sets

for each $x \in (T \cup \text{EOF} \cup \varepsilon)$

$FIRST(x) \leftarrow \{x\}$

for each $A \in \text{NT}$, $FIRST(A) \leftarrow \emptyset$

while (*FIRST* sets are still changing) do

for each $p \in P$, of the form $X \rightarrow Y_1 Y_2 \dots Y_k$ do

temp $\leftarrow FIRST(Y_1) - \{ \varepsilon \}$

$i \leftarrow 1$

while ($i \leq k-1$ and $\varepsilon \in FIRST(Y_i)$)

temp \leftarrow temp \cup ($FIRST(Y_{i+1}) - \{ \varepsilon \}$)

$i \leftarrow i + 1$

end // while loop

if $i == k$ and $\varepsilon \in FIRST(Y_k)$

then temp \leftarrow temp \cup $\{ \varepsilon \}$

$FIRST(X) \leftarrow FIRST(X) \cup$ temp

end // if - then

end // for loop

end // while loop

ε complicates matters

If $FIRST(Y_1)$ contains ε , then we need to add $FIRST(Y_2)$ to rhs, and ...

If the entire rhs can go to ε , then we add ε to $FIRST(\text{lhs})$

Computing *FIRST* sets

for each $x \in (T \cup \text{EOF} \cup \varepsilon)$

$\mathbf{FIRST}(x) \leftarrow \{x\}$

for each $A \in \mathbf{NT}$, $\mathbf{FIRST}(A) \leftarrow \emptyset$

while (*FIRST* sets are still changing) do

for each $p \in P$, of the form $X \rightarrow Y_1 Y_2 \dots Y_k$ do

temp $\leftarrow \mathbf{FIRST}(Y_1) - \{ \varepsilon \}$

$i \leftarrow 1$

while ($i \leq k-1$ and $\varepsilon \in \mathbf{FIRST}(Y_i)$)

temp $\leftarrow \text{temp} \cup (\mathbf{FIRST}(Y_{i+1}) - \{ \varepsilon \})$

$i \leftarrow i + 1$

end // while loop

if $i == k$ and $\varepsilon \in \mathbf{FIRST}(Y_k)$

then temp $\leftarrow \text{temp} \cup \{ \varepsilon \}$

$\mathbf{FIRST}(X) \leftarrow \mathbf{FIRST}(X) \cup \text{temp}$

end // if - then

end // for loop

end // while loop

Outer loop is monotone
increasing for *FIRST* sets
 $\Rightarrow |T \cup \mathbf{NT} \cup \text{EOF} \cup \varepsilon|$ is
bounded, so it terminates

Example

Consider the SheepNoise grammar and its *FIRST* sets

Goal	::= SheepNoise
SheepNoise	::= SheepNoise baa baa

baa is a terminal symbol

Clearly, $FIRST(x) = \{baa\}, \forall x \in (T \cup NT)$

Symbol	<i>FIRST</i> Set
Goal	baa
SheepNoise	baa
baa	baa

Computing *FIRST* sets

```
for each  $x \in (T \cup \text{EOF} \cup \varepsilon)$   
   $\mathit{FIRST}(x) \leftarrow \{x\}$   
for each  $A \in \mathit{NT}$ ,  $\mathit{FIRST}(A) \leftarrow \emptyset$ 
```

```
while ( $\mathit{FIRST}$  sets are still changing) do  
  for each  $p \in P$ , of the form  $X \rightarrow Y_1 Y_2 \dots Y_k$  do  
    temp  $\leftarrow \mathit{FIRST}(Y_1) - \{\varepsilon\}$   
     $i \leftarrow 1$   
    while ( $i \leq k-1$  and  $\varepsilon \in \mathit{FIRST}(Y_i)$ )  
      temp  $\leftarrow \text{temp} \cup (\mathit{FIRST}(Y_{i+1}) - \{\varepsilon\})$   
       $i \leftarrow i + 1$   
    end // while loop  
    if  $i == k$  and  $\varepsilon \in \mathit{FIRST}(Y_k)$   
      then temp  $\leftarrow \text{temp} \cup \{\varepsilon\}$   
       $\mathit{FIRST}(X) \leftarrow \mathit{FIRST}(X) \cup \text{temp}$   
    end // if - then  
  end // for loop  
end // while loop
```

Initialization assigns each *FIRST* set a value

Symbol	<i>FIRST</i> Set
Goal	
SheepNoise	
baa	

Computing *FIRST* sets

for each $x \in (T \cup \text{EOF} \cup \varepsilon)$

$FIRST(x) \leftarrow \{x\}$

for each $A \in \text{NT}$, $FIRST(A) \leftarrow \emptyset$

while (*FIRST* sets are still changing) do

for each $p \in P$, of the form $X \rightarrow Y_1 Y_2 \dots Y_k$ do

temp $\leftarrow FIRST(Y_1) - \{ \varepsilon \}$

$i \leftarrow 1$

while ($i \leq k-1$ and $\varepsilon \in FIRST(Y_i)$)

temp \leftarrow temp \cup ($FIRST(Y_{i+1}) - \{ \varepsilon \}$)

$i \leftarrow i + 1$

end // while loop

if $i == k$ and $\varepsilon \in FIRST(Y_k)$

then temp \leftarrow temp \cup $\{ \varepsilon \}$

$FIRST(X) \leftarrow FIRST(X) \cup$ temp

end // if - then

end // for loop

end // while loop

- | | | | |
|---|-------------------|-----|-----------------------|
| 1 | Goal | ::= | SheepNoise |
| 2 | SheepNoise | ::= | SheepNoise baa |
| 3 | SheepNoise | ::= | baa |

If we visit the rule
in the order 3, 2, 1

Symbol	<i>FIRST</i> Set
Goal	\emptyset
SheepNoise	
baa	{baa}

Computing *FIRST* sets

for each $x \in (T \cup \text{EOF} \cup \varepsilon)$

$FIRST(x) \leftarrow \{x\}$

for each $A \in \text{NT}$, $FIRST(A) \leftarrow \emptyset$

while (*FIRST* sets are still changing) do

for each $p \in P$, of the form $X \rightarrow Y_1 Y_2 \dots Y_k$ do

temp $\leftarrow FIRST(Y_1) - \{ \varepsilon \}$

$i \leftarrow 1$

while ($i \leq k-1$ and $\varepsilon \in FIRST(Y_i)$)

temp $\leftarrow \text{temp} \cup (FIRST(Y_{i+1}) - \{ \varepsilon \})$

$i \leftarrow i + 1$

end // while loop

if $i == k$ and $\varepsilon \in FIRST(Y_k)$

then temp $\leftarrow \text{temp} \cup \{ \varepsilon \}$

$FIRST(X) \leftarrow FIRST(X) \cup \text{temp}$

end // if - then

end // for loop

end // while loop

- 1 Goal ::= SheepNoise
- 2 SheepNoise ::= SheepNoise baa
- 3 SheepNoise ::= baa

If we visit the rule
in the order 3, 2, 1

Symbol	<i>FIRST</i> Set
Goal	
SheepNoise	{baa}
baa	{baa}

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Where LP is (and RP is)

If we visit the rules
in order 4, 3, 2, 1

⇒

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset		
List	\emptyset		
Pair	\emptyset		
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

An Example

Consider the simplest parentheses grammar

- 1 Goal ::= List
- 2 List ::= Pair List
- 3 | ϵ
- 4 **Pair ::= LP List RP**

Where LP is (and RP is)

If we visit the rules
in order 4, 3, 2, 1

⇒

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset		
List	\emptyset		
Pair	\emptyset	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Where LP is (and RP is)

If we visit the rules
in order 4, 3, 2, 1

⇒

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset		
List	\emptyset	<u>LP</u> , ϵ	
Pair	\emptyset	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

An Example

Consider the simplest parentheses grammar

- 1 **Goal ::= List**
- 2 List ::= Pair List
- 3 | ϵ
- 4 Pair ::= LP List RP

Where LP is (and RP is)

If we visit the rules
in order 4, 3, 2, 1

⇒

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset	<u>LP</u> , ϵ	
List	\emptyset	<u>LP</u> , ϵ	
Pair	\emptyset	<u>LP</u>	
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

An Example

Consider the simplest parentheses grammar

- 1 Goal ::= List
- 2 List ::= Pair List
- 3 | ϵ
- 4 **Pair ::= LP List RP**

Where LP is (and RP is)

If we visit the rules
in order 4, 3, 2, 1

⇒

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset	<u>LP</u> , ϵ	
List	\emptyset	<u>LP</u> , ϵ	
Pair	\emptyset	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Where LP is (and RP is)

If we visit the rules
in order 4, 3, 2, 1

⇒

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset	<u>LP</u> , ϵ	
List	\emptyset	<u>LP</u> , ϵ	<u>LP</u> , ϵ
Pair	\emptyset	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

An Example

Consider the simplest parentheses grammar

- 1 **Goal ::= List**
- 2 List ::= Pair List
- 3 | ϵ
- 4 Pair ::= LP List RP

Where LP is (and RP is)

If we visit the rules
in order 4, 3, 2, 1

⇒

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset	<u>LP</u> , ϵ	<u>LP</u> , ϵ
List	\emptyset	<u>LP</u> , ϵ	<u>LP</u> , ϵ
Pair	\emptyset	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

An Example

Consider the simplest parentheses grammar

1	Goal ::= List
2	List ::= Pair List
3	ϵ
4	Pair ::= <u>LP</u> List <u>RP</u>

- Iteration 1 adds LP to *FIRST*(Pair) and LP, ϵ to *FIRST*(List) and *FIRST*(Goal)
 \Rightarrow If we take them in rule order 4, 3, 2, 1
- Algorithm reaches fixed point

FIRST Sets

Symbol	<i>Initial</i>	1 st	2 nd
Goal	\emptyset	<u>LP</u> , ϵ	<u>LP</u> , ϵ
List	\emptyset	<u>LP</u> , ϵ	<u>LP</u> , ϵ
Pair	\emptyset	<u>LP</u>	<u>LP</u>
LP	<u>LP</u>	<u>LP</u>	<u>LP</u>
RP	<u>RP</u>	<u>RP</u>	<u>RP</u>
EOF	EOF	EOF	EOF

FIRST and FOLLOW Sets

FOLLOW(A):

For $A \in \mathbf{NT}$, define **FOLLOW(A)** as the set of tokens that can occur immediately after A in a valid sentential form.

FOLLOW set is defined over the set of non-terminal symbols, **NT**.

Follow Set Construction

To Build FOLLOW(X) for non-terminal X :

- Place EOF in FOLLOW(<start>)
- For each X as a non-terminal, initialize FOLLOW(X) to \emptyset

Iterate until no more terminals can be added to any FOLLOW(X):

For each rule p in the grammar

If p is of the form $A ::= \alpha B \beta$, then

if $\varepsilon \in FIRST(\beta)$

Place $\{FIRST(\beta) - \varepsilon, FOLLOW(A)\}$ in FOLLOW(B)

else

Place $\{FIRST(\beta)\}$ in FOLLOW(B)

If p is of the form $A ::= \alpha B$, then

Place FOLLOW(A) in FOLLOW(B)

End iterate

Computing *FOLLOW* Sets

for each $A \in \mathbf{NT}$

$\mathbf{FOLLOW}(A) \leftarrow \emptyset$

$\mathbf{FOLLOW}(S) \leftarrow \{ \mathbf{EOF} \}$

while (*FOLLOW* sets are still changing) do

for each $p \in P$, of the form $A \rightarrow B_1B_2\dots B_k$ do

Don't add ϵ

TRAILER $\leftarrow \mathbf{FOLLOW}(A)$

for $i \leftarrow k$ down to 1

if $B_i \in \mathbf{NT}$ then // domain checking

$\mathbf{FOLLOW}(B_i) \leftarrow \mathbf{FOLLOW}(B_i) \cup \text{TRAILER}$

if $\epsilon \in \mathbf{FIRST}(B_i)$ // add right context

TRAILER $\leftarrow \text{TRAILER} \cup (\mathbf{FIRST}(B_i) - \{ \epsilon \})$

else TRAILER $\leftarrow \mathbf{FIRST}(B_i)$ // no $\epsilon \Rightarrow$ truncate the right context

else TRAILER $\leftarrow \{ B_i \}$ // $B_i \in \mathbf{T} \Rightarrow$ only 1 symbol

To build *FOLLOW* sets, we need *FIRST* sets

Computing *FOLLOW* Sets

For a production $A \rightarrow B_1B_2 \dots B_k$:

- It works its way backward through the production:
 $B_k, B_{k-1}, \dots B_1$
- It builds the *FOLLOW* sets for the rhs symbols,
 $B_1, B_2, \dots B_k$, not A
- In the absence of ε , *FOLLOW*(B_i) is just *FIRST*(B_{i+1})
 - As always, ε makes the algorithm more complex

To handle ε , the algorithm keeps track of the first word in the trailing right context as it works its way back through rhs: $B_k, B_{k-1}, \dots B_1$

An Example

Consider the simplest parentheses grammar

1	Goal ::= List
2	List ::= Pair List
3	ϵ
4	Pair ::= <u>LP</u> List <u>RP</u>

Symbol	<i>Initial</i>
Goal	EOF
List	\emptyset
Pair	\emptyset

Initial Values:

- Goal, List and Pair are set to \emptyset
- Goal is then set to { **EOF** }

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st
Goal	EOF	
List	\emptyset	
Pair	\emptyset	

Iteration 1:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st
Goal	EOF	EOF
List	\emptyset	EOF
Pair	\emptyset	

Iteration 1:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st
Goal	EOF	EOF
List	\emptyset	EOF
Pair	\emptyset	EOF, LP

Iteration 1:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st
Goal	EOF	EOF
List	\emptyset	EOF, RP
Pair	\emptyset	EOF, LP

Iteration 1:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st
Goal	EOF	EOF
List	\emptyset	EOF, RP
Pair	\emptyset	EOF, LP

Iteration 1:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st	2 nd
Goal	EOF	EOF	
List	\emptyset	EOF, RP	
Pair	\emptyset	EOF, LP	

Iteration 2:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- 1 **Goal ::= List**
- 2 List ::= Pair List
- 3 | ϵ
- 4 Pair ::= LP List RP

Symbol	<i>Initial</i>	1 st	2 nd
Goal	EOF	EOF	EOF
List	\emptyset	EOF, RP	EOF, RP
Pair	\emptyset	EOF, LP	

Iteration 2:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st	2 nd
Goal	EOF	EOF	EOF
List	\emptyset	EOF, RP	EOF, RP
Pair	\emptyset	EOF, LP	EOF, RP, LP

Iteration 2:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

- | | |
|---|-----------------------------------|
| 1 | Goal ::= List |
| 2 | List ::= Pair List |
| 3 | ϵ |
| 4 | Pair ::= <u>LP</u> List <u>RP</u> |

Symbol	<i>Initial</i>	1 st	2 nd
Goal	EOF	EOF	EOF
List	\emptyset	EOF, RP	EOF, RP
Pair	\emptyset	EOF, LP	EOF, RP, LP

Iteration 2:

If we visit the rules
in order 1, 2, 3, 4

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

An Example

Consider the simplest parentheses grammar

1	Goal ::= List
2	List ::= Pair List
3	ϵ
4	Pair ::= <u>LP</u> List <u>RP</u>

Symbol	<i>Initial</i>	1 st	2 nd
Goal	EOF	EOF	EOF
List	\emptyset	EOF, RP	EOF, RP
Pair	\emptyset	EOF, LP	EOF, RP, LP

Iteration 2:

- Production 1 adds nothing new
- Production 2 adds RP to *FOLLOW*(Pair)
from *FOLLOW*(List), $\epsilon \in \text{FIRST}(\text{List})$
- Production 3 does nothing
- Production 4 adds nothing new

Symbol	<i>FIRST</i> Set
Goal	<u>LP</u> , ϵ
List	<u>LP</u> , ϵ
Pair	<u>LP</u>
LP	<u>LP</u>
RP	<u>RP</u>
EOF	EOF

Iteration 3 produces the same result \Rightarrow reached a fixed point

Building Top-down Parsers

Building the *FIRST+* set

- Need a *FIRST+* set for every rule

Symbol	<i>FIRST</i>	<i>FOLLOW</i>
Goal	<u>LP</u> , ϵ	EOF
List	<u>LP</u> , ϵ	EOF, RP
Pair	<u>LP</u>	EOF, RP, LP
LP	<u>LP</u>	-
RP	<u>RP</u>	-
EOF	EOF	-

1	Goal ::= List
2	List ::= Pair List
3	ϵ
4	Pair ::= <u>LP</u> List <u>RP</u>

Rule	<i>FIRST+</i>
1	EOF, LP
2	LP
3	EOF, RP
4	LP

Building Top-down Parsers

Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

	<i>Rule</i>	<i>FIRST+</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>
1	Goal ::= List	1	EOF, LP	Goal	1	1
2	List ::= Pair List	2	LP	List		
3	ϵ	3	EOF, RP	Pair		
4	Pair ::= <u>LP</u> List <u>RP</u>	4	LP			

Building Top-down Parsers

Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

	<i>Rule</i>	<i>FIRST+</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>	
1	Goal ::= List	1	EOF, RP	Goal	1		1
2	List ::= Pair List	2	LP	List	2	3	3
3	ϵ	3	EOF, RP	Pair			
4	Pair ::= <u>LP</u> List <u>RP</u>	4	LP				

Building Top-down Parsers

Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

	<i>Rule</i>	<i>FIRST+</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>	
1	Goal ::= List	1	EOF, RP	Goal	1		1
2	List ::= Pair List	2	LP	List	2	3	3
3	ϵ	3	EOF, RP	Pair	4		
4	Pair ::= <u>LP</u> List <u>RP</u>	4	LP				

Building Top-down Parsers

Building the complete parse table

- Need a row for every **NT** and a column for every **T**
- Need an interpreter for the table (skeleton parser)

	<i>Rule</i>	<i>FIRST+</i>		<i>LP</i>	<i>RP</i>	<i>EOF</i>
1	Goal ::= List	EOF, RP	Goal	1		1
2	List ::= Pair List	LP	List	2	3	3
3	ϵ	EOF, RP	Pair	4		
4	Pair ::= <u>LP</u> List <u>RP</u>	LP				

Review: Table Driven LL(1) Parsing

Input: a string w and a parsing table M for G

push eof

push *Start* Symbol

token \leftarrow *next_token*()

$X \leftarrow$ top-of-stack

repeat

 if X is a terminal then

 if $X ==$ token then

 pop X

 token \leftarrow *next_token*()

 else error()

 else /* X is a non-terminal */

 if $\mathbf{M}[X, \text{token}] == X \rightarrow Y_1 Y_2 \dots Y_k$ then

 pop X

 push Y_k, Y_{k-1}, \dots, Y_1

 else error()

$X \leftarrow$ top-of-stack

until $X = \text{EOF}$

if token \neq EOF then error()

	<i>LP</i>	<i>RP</i>	<i>EOF</i>
Goal	1		1
List	2	3	3
Pair	4		

M is the parse table

Next Lecture

Things to do:

- Start programming in C.
- Read Scott, Chapter 3.1 - 3.3; ALSU 7.1
- Read Scott, Chapter 8.1 - 8.2; ALSU 7.1 - 7.3