CS 314 Principles of Programming Languages

Lecture 21: Parallelism and Dependence Analysis

Zheng (Eddy) Zhang

Rutgers University

April 23, 2018

Class Information

- Homework 7 is released.
- Homework 8 (our last homework) will be released this Wednesday.
- Project 2 is due this Thursday.
- Please pick up your midterm exam if you haven't.
  You can pick them up at my office hour or any TA's office hour.

Programming with Concurrency

- A PROCESS or THREAD is a potentially-active execution context
- Classic *von Neumann* model of computing has single thread of control, however parallel programs have more than one
- A process or thread can be thought of as

  An abstraction of a physical PROCESSOR
- Processes/Threads can come from
  - Multiple CPUs
  - Kernel-level multiplexing of single physical machine
  - Language or library level multiplexing of kernel-level abstraction
- They can run
  - In true **parallel**
  - Unpredictably interleaved
  - Run-until-block

Dependence and Parallelization

Dependence analysis is fundamental to parallelization analysis

Dependence relation: all *task–to–task* execution orderings that must be preserved if the meaning of the program is to remain the same.

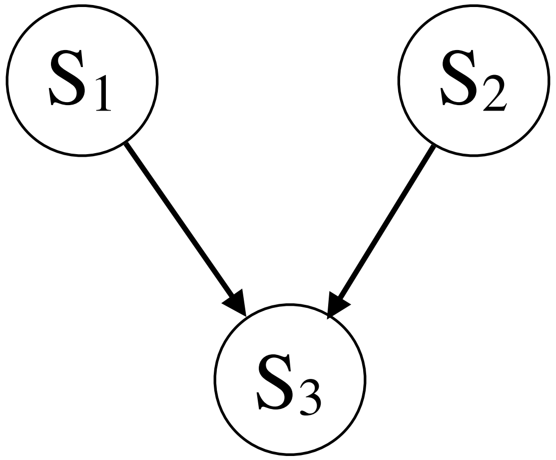
The dependence relation can be modeled as a directed graph such that if A → B, the result of task A is required for the processing of task B

Example:

S_1: pi = 3.14

S_2: R = 5

S_3: Area = pi * R^2

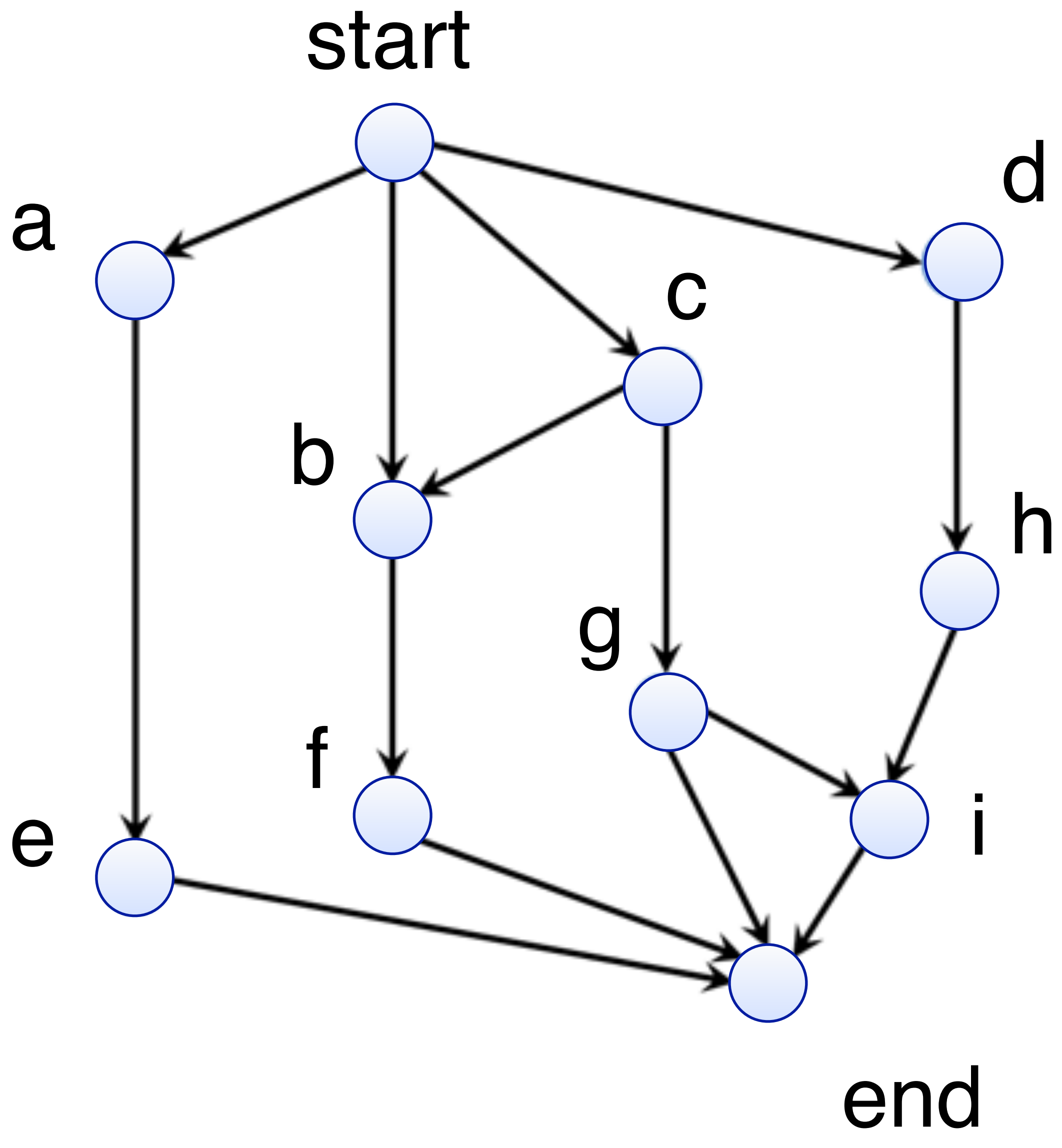


Statement-level dependence graph

Dependence Graph

- Directed acyclic graph (DAG)
- A node represents a task
- A directed edge represents precedence constraint

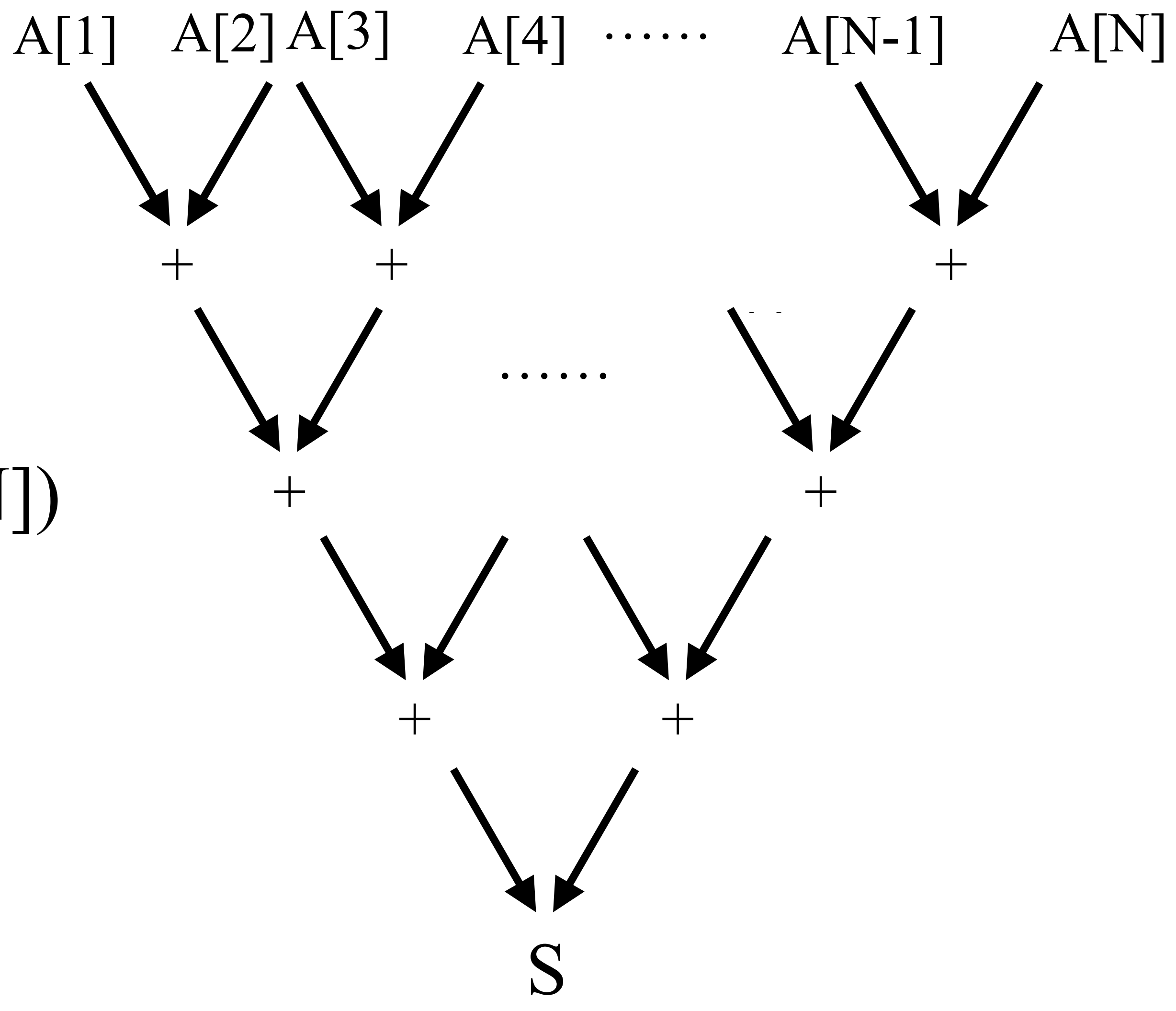
DAG example 1:



Dependence Graph

- Directed acyclic graph (DAG)
- A node represents a task
- A directed edge represents precedence constraint

DAG example 2:



S = sum(A[1], A[2], …, A[N])

Scheduling a DAG

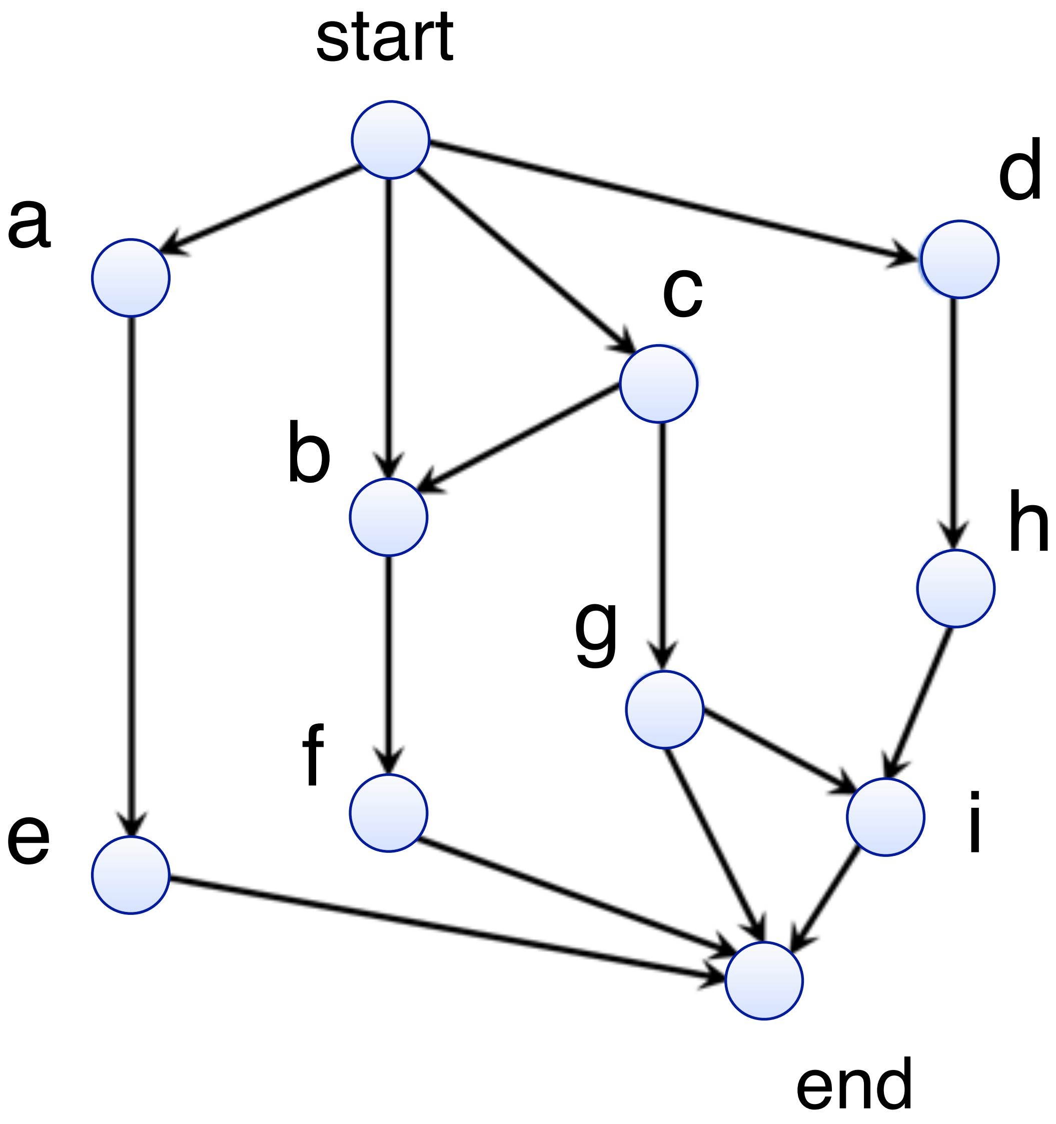
T_p: time to perform computation with *p* processors

- T_1 : **work** (total # operations)
- T_∞ : **critical path** or **span**

$$T_p \geq \frac{T_1}{p}, \qquad T_p \geq T_\infty$$

Maximum parallelism: T_1 / T_∞

Linear speedup: $\frac{T_p}{T_1} = \Theta(p)$



Scheduling a DAG

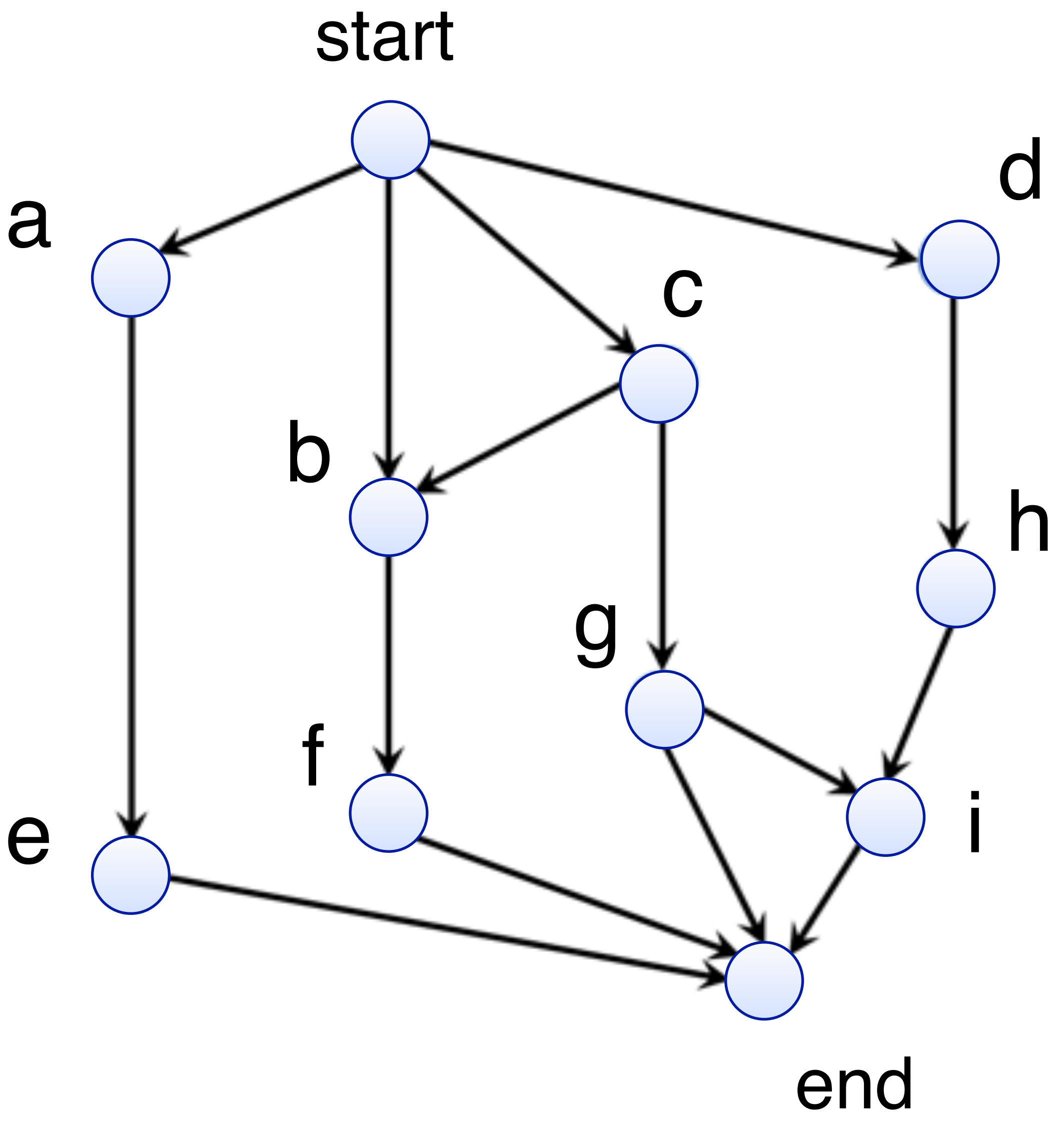
T_p: time to perform computation with *p* processors

- T_1 : **work** (total # operations)
- T_∞ : **critical path** or **span**

$$T_p \geq \frac{T_1}{p}, \qquad T_p \geq T_\infty$$

Maximum parallelism: T_1 / T_∞

Linear speedup: $\frac{T_p}{T_1} = \Theta(p)$



$T_1 = ?$

Scheduling a DAG

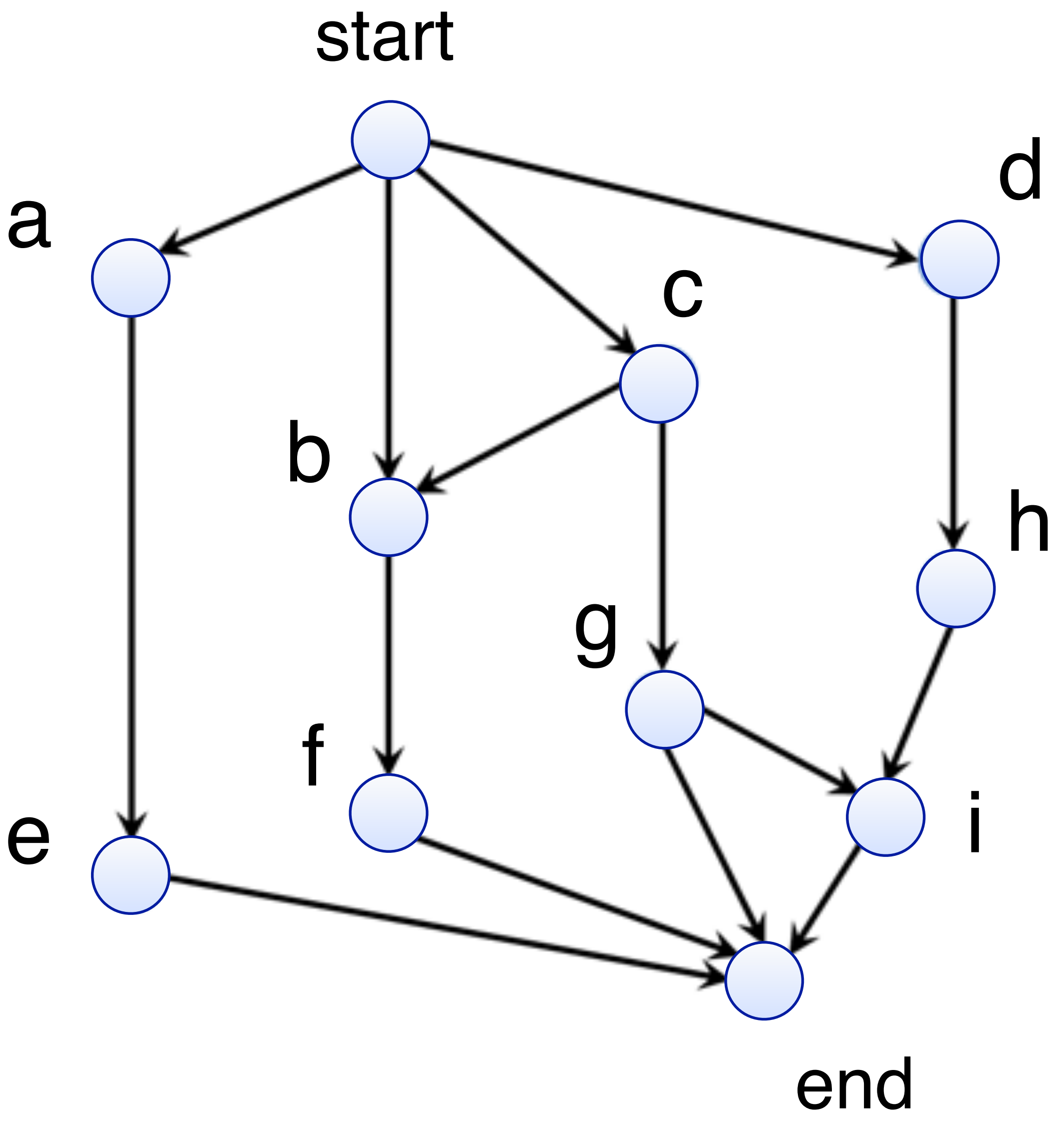
T_p: time to perform computation with *p* processors

- T_1 : **work** (total # operations)
- T_∞ : **critical path** or **span**

$$T_p \geq \frac{T_1}{p}, \qquad T_p \geq T_\infty$$

Maximum parallelism: T_1 / T_∞

Linear speedup: $\frac{T_p}{T_1} = \Theta(p)$



$T_\infty = ?$

Scheduling a DAG

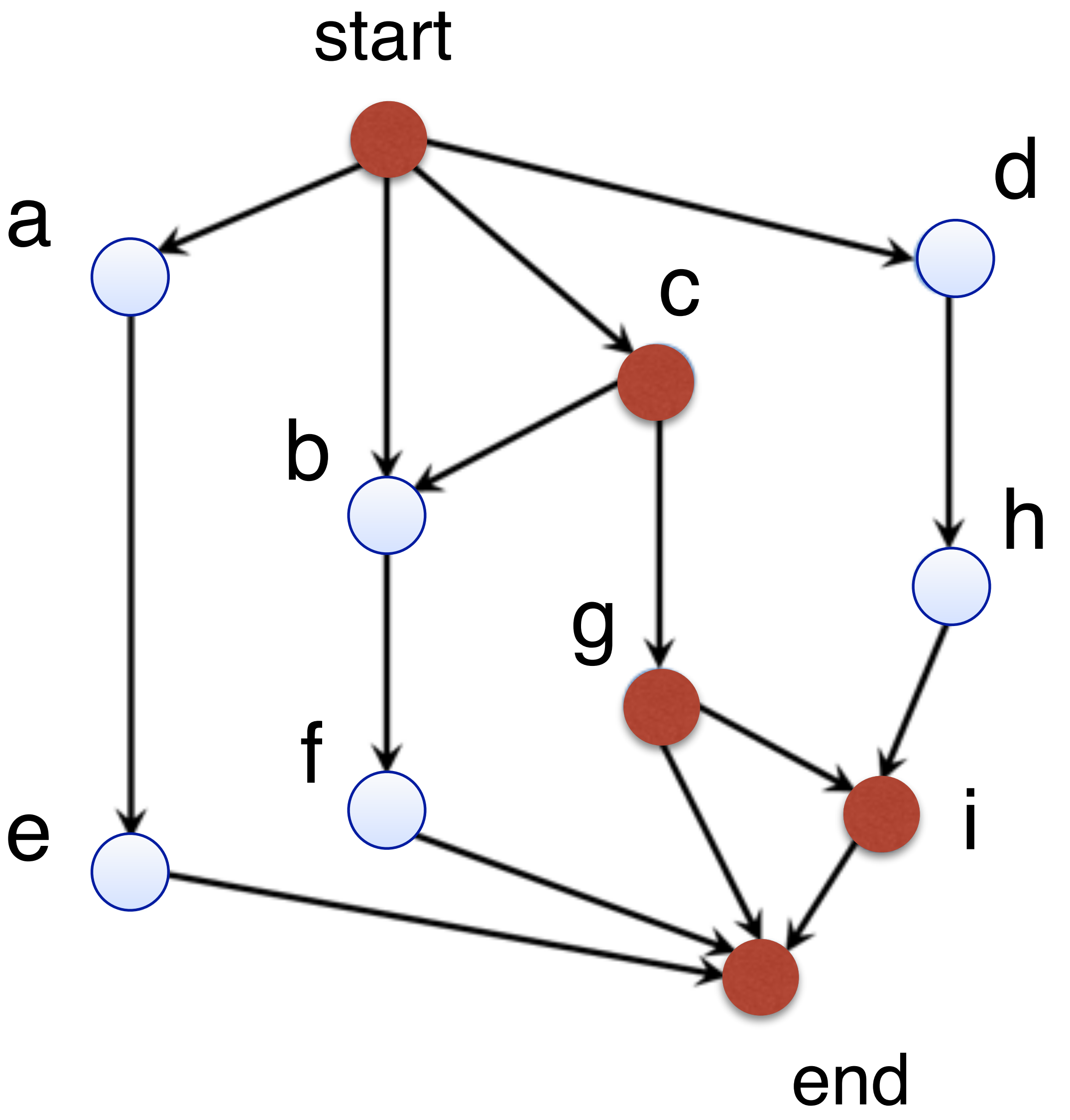
T_p: time to perform computation with *p* processors

- T_1 : **work** (total # operations)
- T_∞ : **critical path** or **span**

$$T_p \geq \frac{T_1}{p}, \qquad T_p \geq T_\infty$$

Maximum parallelism: T_1 / T_∞

Linear speedup: $\frac{T_p}{T_1} = \Theta(p)$



$T_\infty = ?$

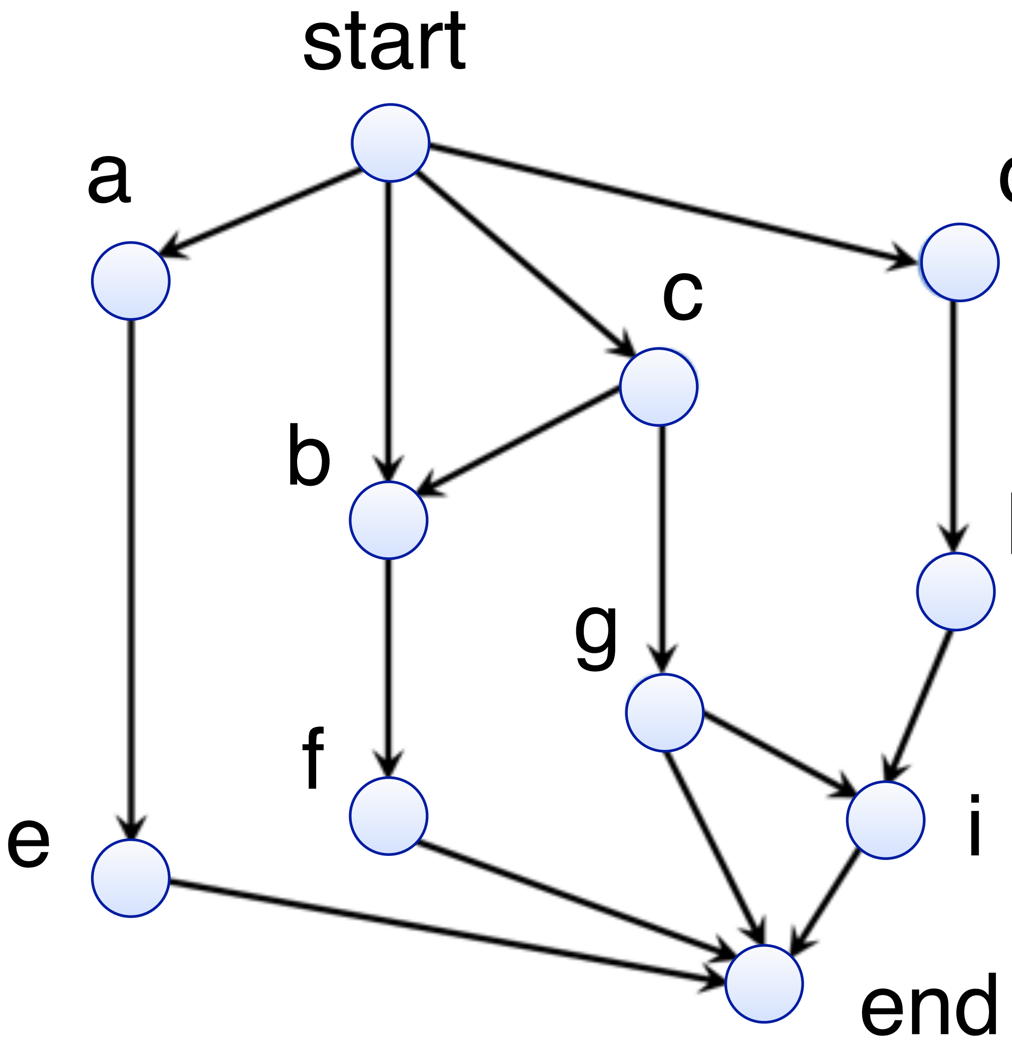
Computing Critical Path

Compute the earliest start time of each node

- Keep a value called S(n) associated with each node n
- For each node n

  S(n) is the maximum of { S(p) + 1 }, for all p ∈ pred(n)

Assuming a task takes 1 unit time



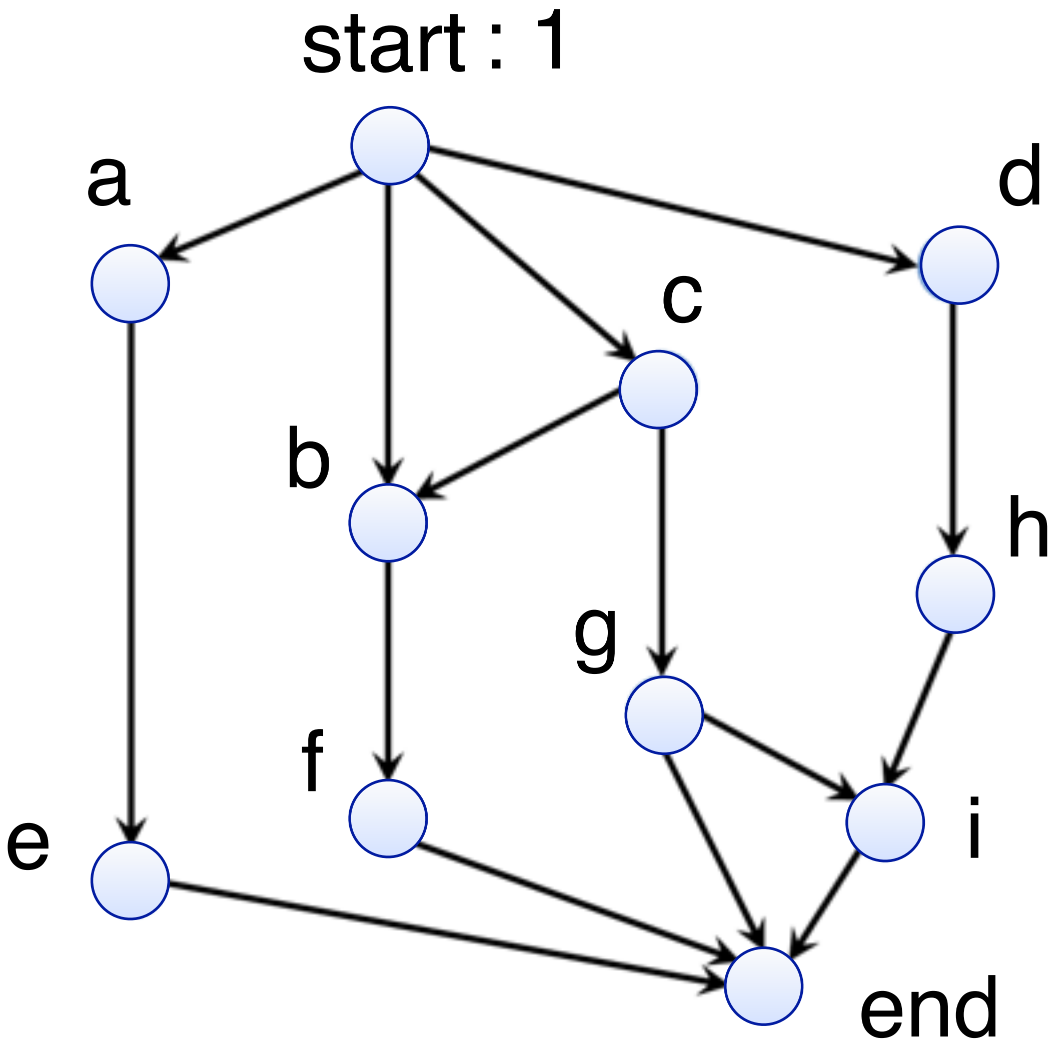
Computing Critical Path

Compute the earliest start time of each node

- Keep a value called S(n) associated with each node n
- For each node *n*

  S(n) is the maximum of { S(p) + 1 }, for all p $\in$ pred(n)

Assuming a task takes 1 unit time



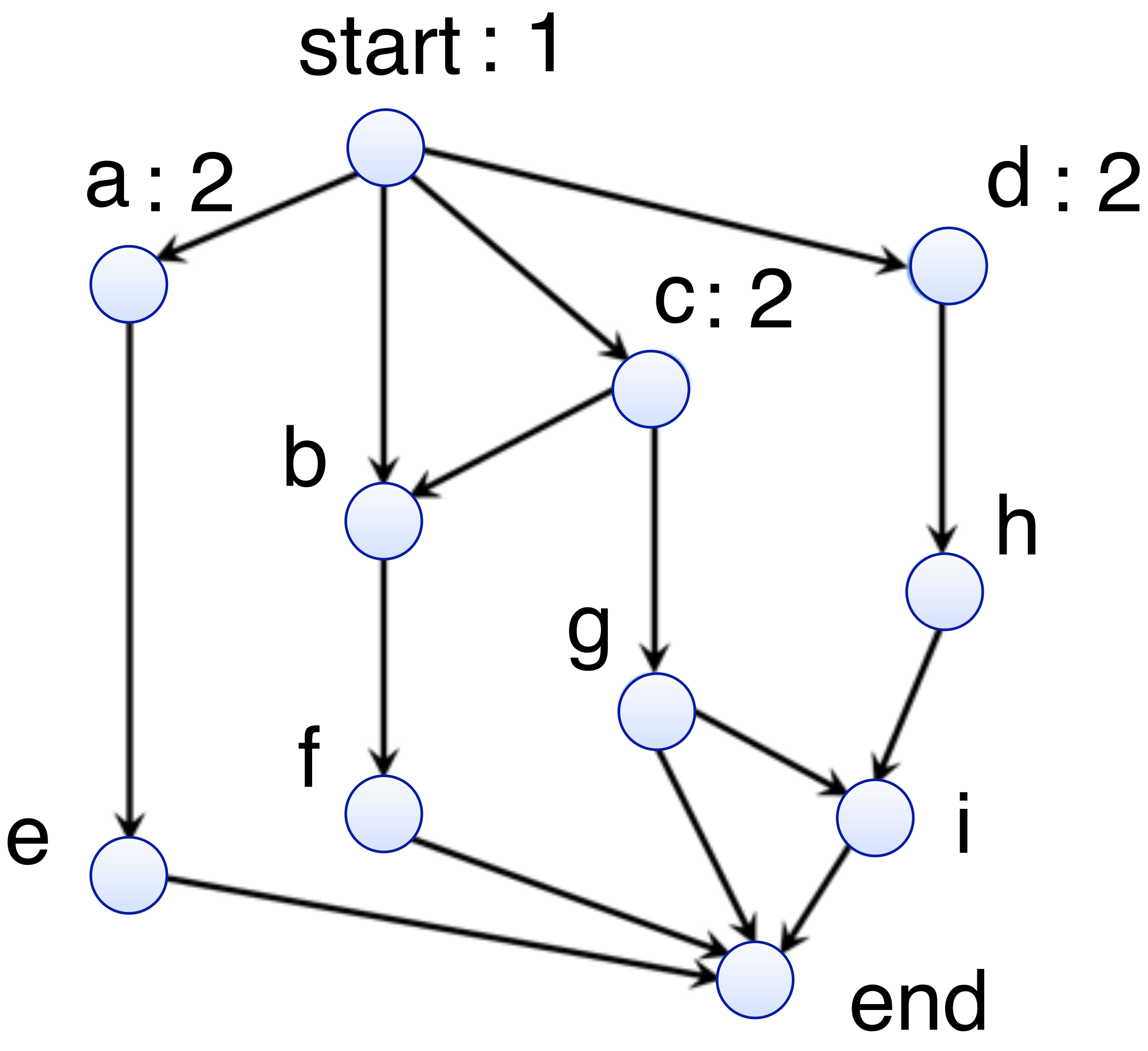
Computing Critical Path

Compute the earliest start time of each node

- Keep a value called S(n) associated with each node n
- For each node *n*

  S(n) is the maximum of { S(p) + 1 }, for all p ∈ pred(n)

Assuming a task takes 1 unit time



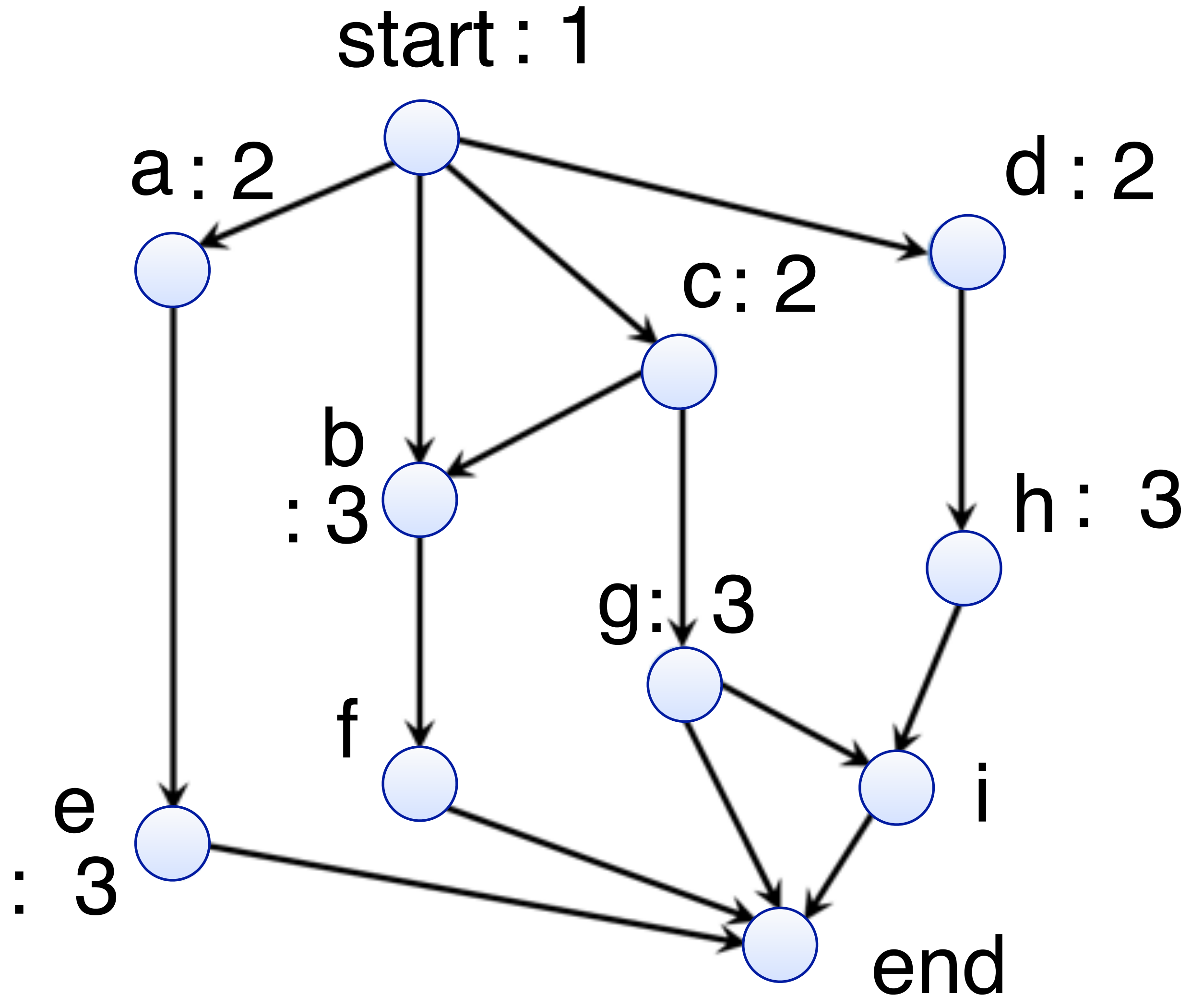
Computing Critical Path

Compute the earliest start time of each node

- Keep a value called S(n) associated with each node n
- For each node *n*

  S(n) is the maximum of { S(p) + 1 }, for all p $\in$ pred(n)

Assuming a task takes 1 unit time



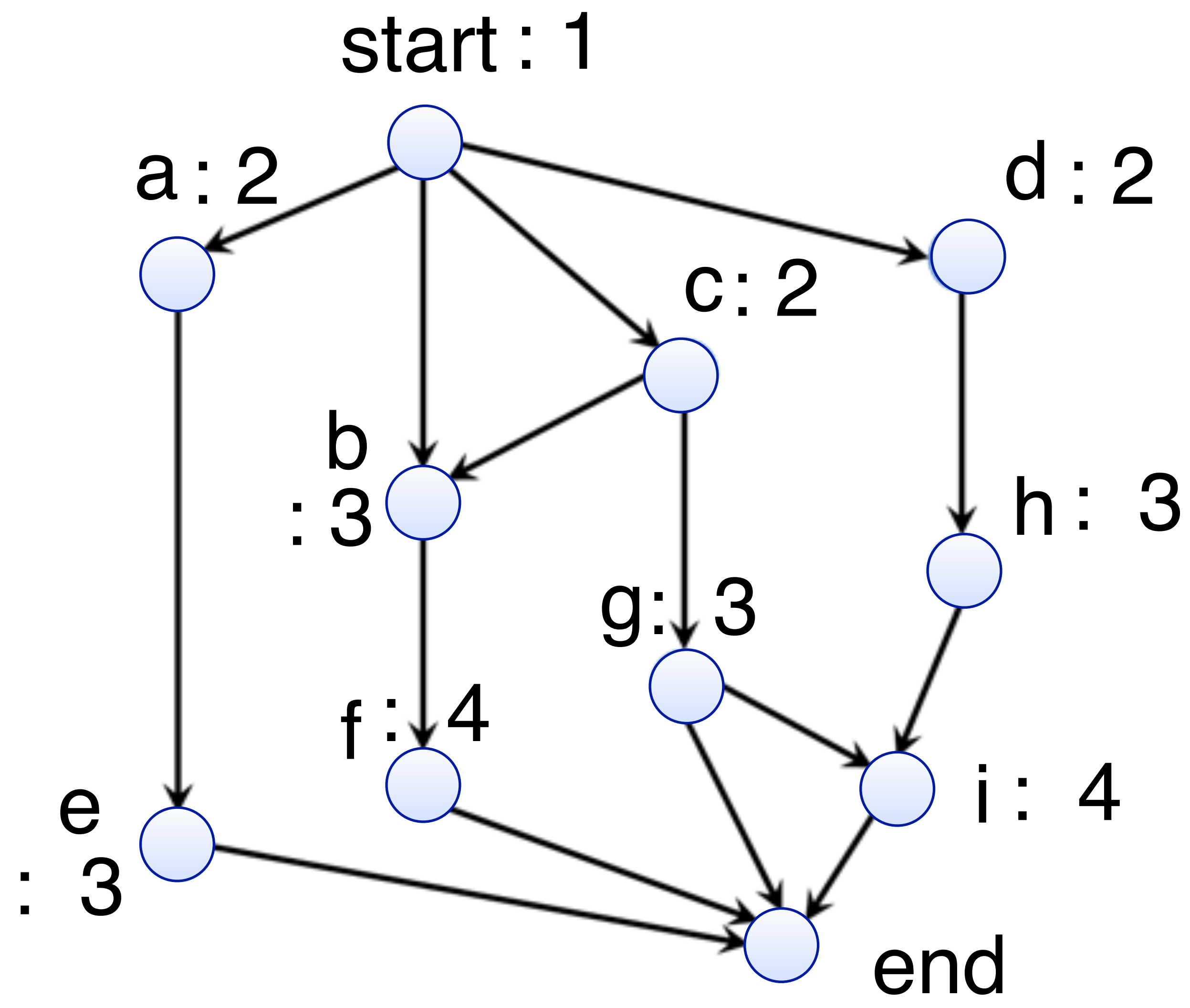
Computing Critical Path

Compute the earliest start time of each node

- Keep a value called S(n) associated with each node n
- For each node n

  S(n) is the maximum of { S(p) + 1 }, for all p $\in$ pred(n)

Assuming a task takes 1 unit time



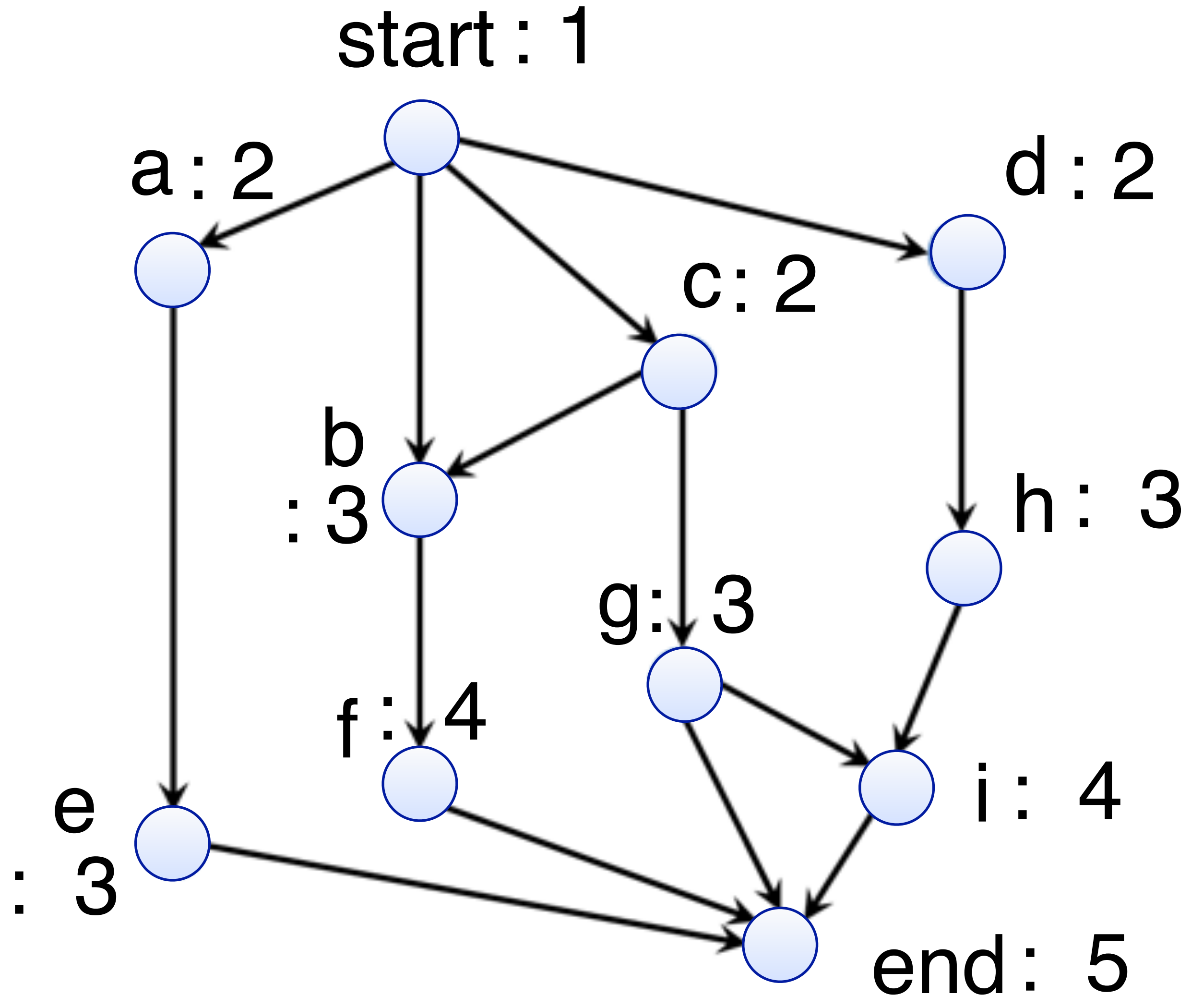
Computing Critical Path

Compute the earliest start time of each node

- Keep a value called S(n) associated with each node n
- For each node *n*

  S(n) is the maximum of { S(p) + 1 }, for all p ∈ pred(n)

Assuming a task takes 1 unit time



List Scheduling

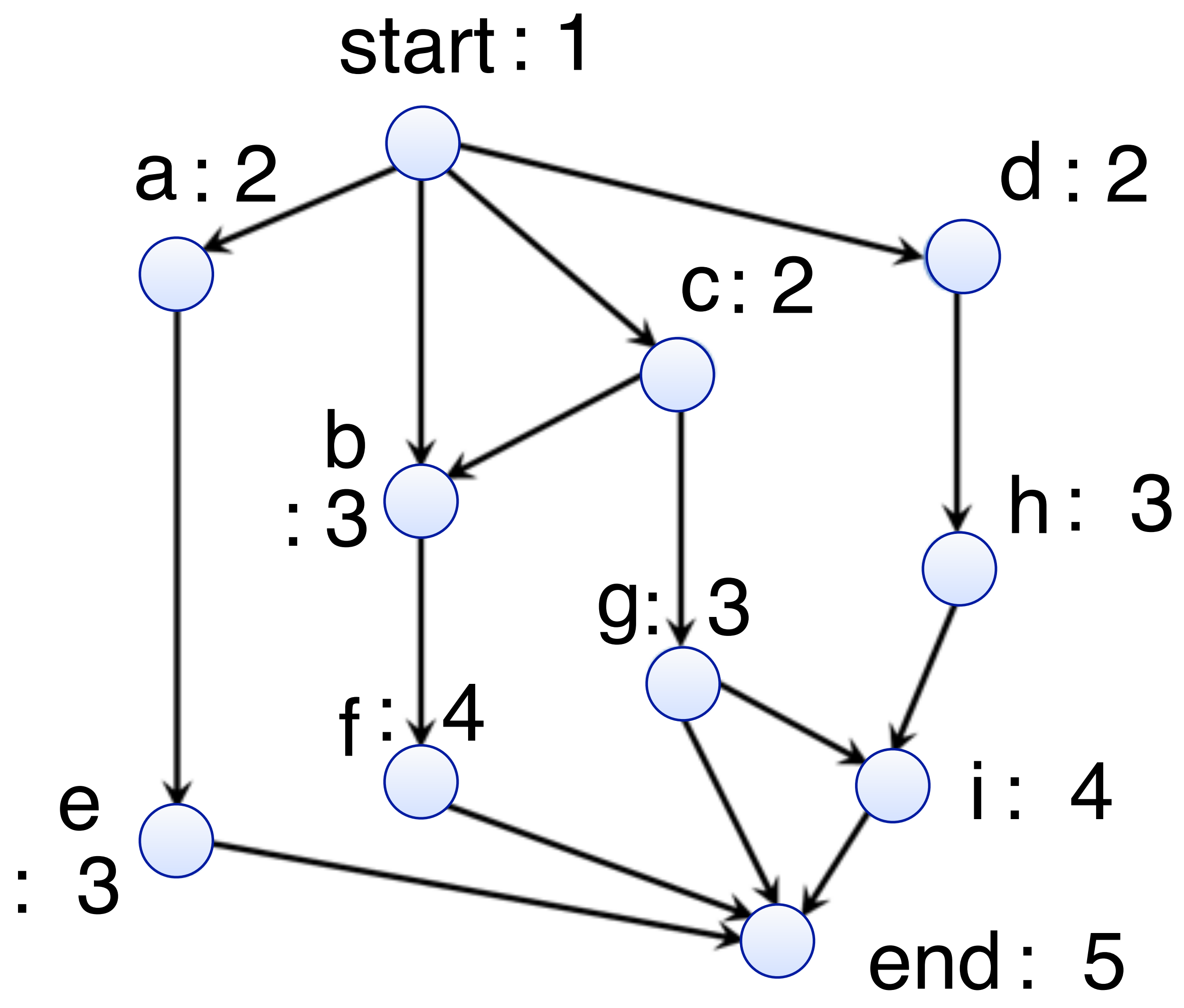
Based on if the dependence constraints have been resolved

- Schedule the nodes that are ready at every time tick
- A completed operation at the end of one time step can lead to more ready operations at next time tick

Four threads T1, T2, T3, T4

	1	2	3	4	5
T1	start	a	b	f	end
T2		c	e	i	
T3		d	g		
T4			h		

Assuming a task takes 1 unit time



Automatic Parallelization

We will use **loop analysis** as an example to describe automatic dependence analysis and parallelization.

Assumptions:

1. We only have scalar and subscripted variables (no pointers and no control dependence) for loop dependence analysis.
2. We focus on ***affine loops***: both loop bounds and memory references are affine functions of loop induction variables.

A function $f(x_1, x_2, ..., x_n)$ is **affine** if it is in such a form:

$\mathbf{f} = c_0 + c_1 * \boldsymbol{x_1} + c_2 * \boldsymbol{x_2} + \ldots + c_n * \boldsymbol{x_n}$, where c_i are all constants

Affine Loops

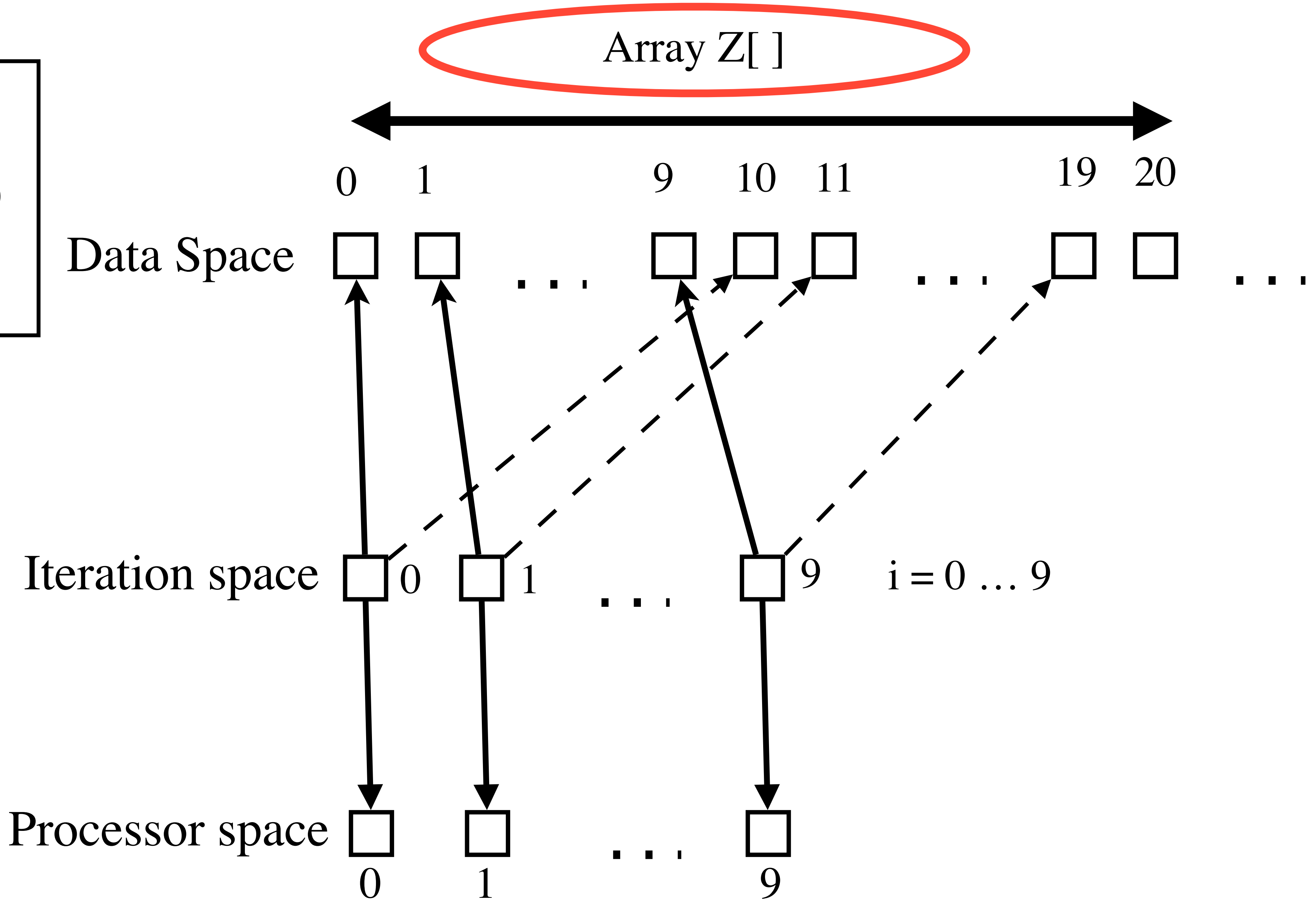
Three spaces

- Iteration space
  - The set of dynamic execution instances
  - i.e. the set of value vectors taken by loop indices
  - A *k*-dimensional space for a *k*-level loop nest
- Data space
  - The set of array elements accessed
  - An *n*-dimensional space for an *n*-dimensional array
- Processor space
  - The set of processors in the system
  - In analysis, we may pretend there are unbounded # of virtual processors

Three Spaces

- **Iteration space, data space, and processor space**

```
float Z[100];
for (i=0; i<10; i++)
 Z[i+10] = Z[i];
```



Assuming one task is one loop iteration, what is the maximum parallelism?

Maximum parallelism: T_1 / T_∞

Dependence Definition

Bernstein's Condition: — There is a data dependence from statement (instance) S_1 to statement S_2 (instance) if

- Both statements (instances) access the same memory locations
- One of them is a write
- There is a run-time execution path from S_1 to S_2

```
float Z[100];
for (i=0; i<10; i++)
  Z[i+10] = Z[i];
```

No dependence across any two loop iterations!

Data Dependence Classifications

"S_2 depends on S_1" — ($S_1\ \delta\ S_2$)

True (flow) dependence

occurs when S1 writes a memory location that S2 later reads (RAW).

Anti dependence

occurs when S1 reads a memory location that S2 later writes (WAR).

Output dependence

occurs when S1 writes a memory location that S2 later writes (WAW).

Input dependence

occurs when S1 reads a memory location that S2 later reads (RAR).

Simple Dependence Testing

- **Examples:**

```
for (i = 1; i <= 100; i++) {
   S1:  A[i] = ...
   S2:  ...= A[i - 1]
}
```

```
float Z[100];
for (i =0; i < 12; i++) {
    S: Z[ i+10 ] = Z[i];
}
```

1. Is there dependence?
2. If so, what type of dependence?
3. From which statement (instance) to which statement (instance)?

Dependence Testing

Single Induction Variable (SIV) Test

- Single loop nest with constant lower (LB) and upper (UB) bound, and step 1.

```
    for i = LB, UB, 1
        ...
    endfor
```

- Two array references as affine function of loop induction variable

```
    for i = LB, UB, 1
R1:     X(a*i + c1) = ...       \\ write
R2:       ... = X(a*i + c2) ...  \\ read
    endfor
```

Question: Is there a true dependence between R1 and R2?

Dependence Testing

```
    for i = LB, UB, 1
R1:     X(a*i + c1) = ...       \\ write
R2:        ... = X(a*i + c2) ...  \\ read
    endfor
```

There is a dependence between R1 and R2 **iff**

$$\exists\, i, i': \ LB \leq i \leq i' \leq UB \text{ and } (a*i+c_1) = (a*i'+c_2)$$

where i and i′ represent two iterations in the iteration space. This means that in both iterations, the same element of array X is accessed.

So let's just solve the equation:

$$(a * i + c_1) = (a * i' + c_2) \quad \Rightarrow \quad (c_1 - c_2)/a = i' - i = \Delta d$$

There is a dependence iff
- Δd is an integer value
- $UB - LB \geq \Delta d \geq 0$

Simple Dependence Testing

- **Examples:**

```
for (i = 1; i <= 100; i++) {
  S1:  A[i] = ...
  S2:  ...= A[i - 1]
}
```

```
float Z[100];
for (i =0; i < 12; i++) {
    S: Z[ i+10 ] = Z[i];
}
```

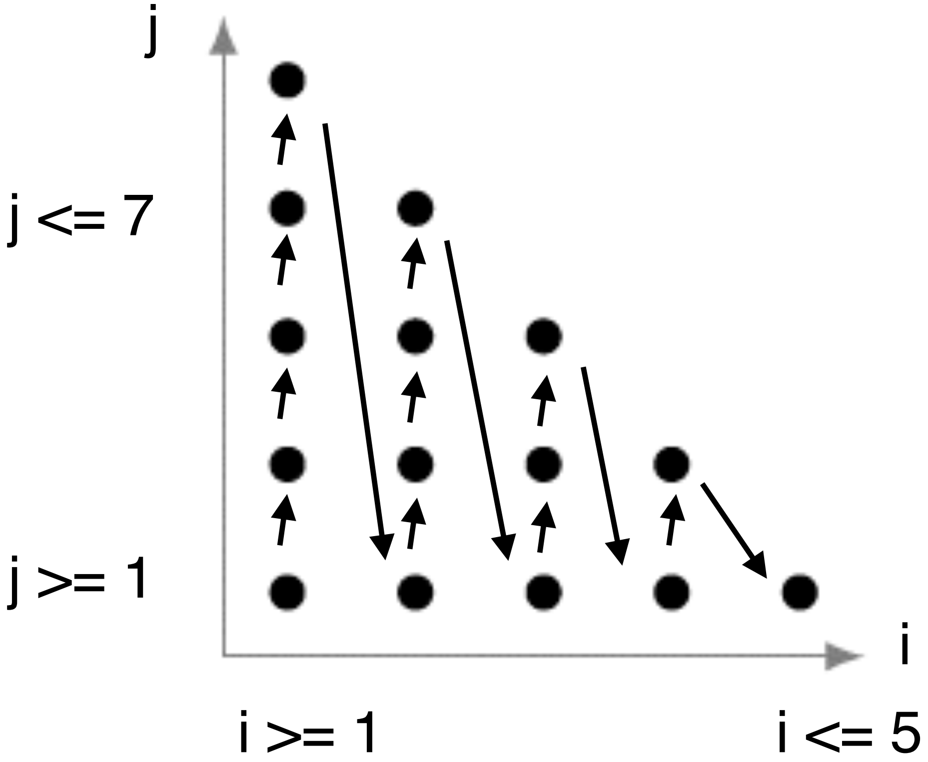
1. Is there dependence?
2. If so, what type of dependence?
3. From which statement (instance) to which statement (instance)?

Lexicographical Order

- Order of sequential loop executions
- Sweeping through the space in an ascending lexicographic order:

  (i, j) <= (i’, j’) iff one of the two conditions is satisfied
  1. i < i’
  2. i = i’ & j <= j’

```
for (i = 1; i <= 5; i++)
   for (j = 1; j <= 6 - i; j++)
      Z[j, i] = 0;
```



Dependence Testing

- **Example:**

```
do I = 1, 99
   do J = 1, 100
      A(I,J) = A(I+1,J) + 1
   end do
end do
```

1. Is there dependence?
2. If so, what type of dependence?
3. From which statement (instance) to which statement (instance)?
4. Which loop (i or j) can be parallelized?

Next Class

Reading:

- ALSU, Chapter 11.1 - 11.3

Review: Dependence Test

Given

```
do $i_1 = L_1,U_1$

  ...

    do $i_n = L_n,U_n$
        S1 :   $A[\ f_1(\ i_1, \ldots, i_n), \ldots, f_m(i_1,\ldots, i_n)\ ] = \ldots$
        S2 :   $\ldots\ = A[\ g_1(i_1, \ldots, i_n), \ldots, g_m(i_1, \ldots, i_n)\ ]$
```

A dependence between statement (instance) S_1 and S_2, denoted $S_1\ \delta\ S_2$, indicates that the S_1 instance, the source, must be executed before S_2 instance, the sink on some iteration of the nest.

Let α & β be a vector of n integers within the ranges of the lower and upper bounds of the n loops.

Does $\exists\ \alpha \leq \beta$ in the loop iteration space, s.t.

$$f_k(\alpha) = g_k(\beta) \qquad \forall k, 1 \leq k \leq m?$$