1 Problem — Finite State Automaton (FSA)

1. Specify the state transition graph of (1) a NFA (which is not a DFA as well) without $\epsilon$ transitions and (2) a DFA that recognizes the following language: “All strings of 0’s and 1’s that, when interpreted as a binary number, are divisible by 4. In other words, value(binary number) modulo 4 = 0.”

2. In addition to the state transition graphs (diagram), give the state transition table and the formal specification of an automaton as the quadrupel $<S, s, F, \Delta>$ for both, your NFA and DFA. Do not include “error” states.

2 Problem — Regular and Context-Free Languages

Are the following languages context-free or not? If yes, specify a context-free grammar in BNF notation that generates the language. If not, give an informal argument.

1. $\{ a^m b^n c^o \ | \ m > 0, n \geq 0, o > 0 \}$, with alphabet $\Sigma = \{a, b, c\}$
2. $\{ a^n b^n c^n \ | \ m > n \geq 0, o > 0 \}$, with alphabet $\Sigma = \{a, b, c\}$
3. $\{ a^n b c^m \ | \ n > 0 \}$, with alphabet $\Sigma = \{a, b, c\}$
4. $\{ a^{2m} b^n \ | \ n \geq 0 \}$, with alphabet $\Sigma = \{a, b\}$
5. $\{ w w^R \ | \ w \in \Sigma^* \text{ and } w^R \text{ is } w \text{ in reverse } \}$, with alphabet $\Sigma = \{a, b\}$
6. $\{ a^n b^m c^n d^m \ | \ n \geq 0, m \geq 0 \}$, with alphabet $\Sigma = \{a, b, c, d\}$
7. $\{ a^n b^m c^n d^m \ | \ n \geq 0, m \geq 0 \}$, with alphabet $\Sigma = \{a, b, c, d\}$
8. $\{ a^n b^m c^n d^m \ | \ n \geq 0, m \geq 0 \}$, with alphabet $\Sigma = \{a, b, c, d\}$
9. $\{ a^n a^n b^n b^n \ | \ n \geq 0 \}$, with alphabet $\Sigma = \{a, b\}$
10. $\{ w \ | \ w \text{ has more than 5 symbols} \}$, with alphabet $\Sigma = \{a, b\}$

Which of the languages are also regular languages, i.e., can be expressed by a regular expression? Prove it by giving the regular expression that specifies the language.
3 Problem — Derivation, Parse Tree, Ambiguity, Precedence & Associativity

A language that is a subset of the language of propositional logic may be defined as follows:

\[
\begin{align*}
&\text{	exttt{<start> ::= <expr>}} \\
&\text{	exttt{<expr> ::= <expr> \lor <expr> |}} \\
&\phantom{\text{	exttt{<expr> ::= <expr> \lor <expr> |}}}\text{<expr> \land <expr> |} \\
&\phantom{\text{<expr> ::= <expr> \lor <expr> | <expr> \land <expr> | <expr> \lor <expr> |}}\text{<expr> \rightarrow <expr> |} \\
&\phantom{\text{<expr> ::= <expr> \lor <expr> | <expr> \land <expr> | <expr> \lor <expr> | <expr> \rightarrow <expr> |}}\text{<const> | <var>}
\end{align*}
\]

\[
\begin{align*}
&\text{	exttt{<const> ::= true | false}} \\
&\text{	exttt{<var> ::= a | b | c | \ldots | z}}
\end{align*}
\]

1. Give a leftmost and a rightmost derivation for the sentence

\[a \land \text{false} \lor b \rightarrow \text{true} .\]

2. Give the corresponding parse trees for the derivations.

3. Give the corresponding abstract syntax tree (AST).

4. Show that the above grammar is ambiguous.

5. Give an unambiguous grammar for the same language that enforces the following precedence and associativity:

- \lor has highest precedence (binds strongest), followed by \land, and then \rightarrow
- \land is left associative, and \rightarrow and \lor are right associative

6. Give the parse tree and AST for your new, unambiguous grammar for the sentence

\[a \land \text{true} \land b \rightarrow \text{false} \lor \text{true} .\]