**CS323** **Exam Topics** **May 6, 2004**

Here is what we covered that could be questioned on the exam. The exam is cumulative, but will definitely focus on the post-midterm material

1. **k-digit arithmetic:**. Know how to represent real numbers in this number system - rounding or chopping - and how to do arithmetic operations. absolute and relative error of a computation.

2. **Nonlinear Equations**.

   (a) Specific methods: bisection, regula-falsi, Newton's, secant, chord

   (b) Fixed Point Iteration: what it does geometrically; its convergence behaviour (contraction mapping principle); its convergence rate.

   (c) accelleration of linear convergence (aitkin, steffanson)

3. **Linear systems:**.

   (a) How to solve them: Gaussian Elimination, Gausss-Jordan Reduction; computation of the inverse; LU factorization. LUP factorization; roundoff errors, pivoting strategies.

   (b) Computational Costs: for each of the above, also know how to count such costs for other operations such as multiplying matrices with goven structure for zeros (ie lower triangular)

4. **Interpolation and Approximation:**

   (a) Taylor Polys: Know how to find the Taylor poly of degree n, the formula for its error, Taylors theorem.

   (b) Interpolation: Know the interpolation theorem and how to find the (unique) n-th degree polynomial that agrees with a function at n+1 given points using Lagranges form, Newtons form, the standard form.

   (c) Know the error formula and how to use it.

   (d) Runges phenomenon: details about the disastrous choice of collocation points

   (e) Tchebycheff Interpolation: (i) how to find it; (ii) Powell's thm; (iii) it is minimax if f is a poly of degree at most n+1.

   (f) (i) set up and solve the normal equations to find the CONTINUOUS least squares approximation in a given basis; (ii) know the connection with the Hilbert matrix (with the monomial basis on [0,1], coefficients of the normal equations are given by the Hilbert matrix) so high cost to find the LSQ approx and with large roundoff. (iii) The remedy is to use an orthogonal basis so the Normal eqns have a diagonal coefficient matrix (small cost to solve, small roundoff). Know the Legendre fns for [-1,1] and how to map them to [a.b]; (iv) set up and solve normal equations for DISCRETE least squares approximation using given datapoints. [NOT RESPONSIBLE FOR WEIGHTED DISCRETE LEAST SQUARES.]

OVER

5. **Integration:**

   (a) DIFFERENTIATION NOT COVERED.

   (b) The Integration Rules: Rectangular rule (k=0), Midpoint rule (k=0), Simpson's rule (k=2), $\text{Gauss}_2$ (k=1) and $\text{Gauss}_3$ (k=2), composite and simple (the $k$ refers to the degree of interpolation on which the rule is based). [we did NOT COVER THE TRAPEZOIDAL RULE]

   (c) The error formulae and how to use them; how many subdivisions needed to achieve a certain error bound; how much error with a given number of subdivisions.