CS442 **March 1, 2005**

TEST 1 - Some Solutions

1. The challenge of this question is to find a known theorem, or fact, which supports the assertion. If you cannot, you may believe the assertion is not generally true, and then search for a conterexample. Constructing a counterexample for the assertion that is still consistent with the given data may actually take some time. If you dont see an immediate reason why the assertion may be false,A good strategy might be to say "I believe this is not generally true, and will come back later to construct a counter-example".

   (a) $P(Y=3) < 1$ is **FALSE**: the constant random variable $Y=3$ is consistent with the given data.

   (b) $P(X \geq 1) \geq \frac{1}{10}$ is **TRUE** by the analog of Markov's inequality. It says that Z is a random variable with $E(Z)=\mu$, $Z \leq B$ with probabiolity 1, and $t \leq \mu$, then

   $$P(Z \geq t) \geq \frac{\mu - t}{B - t}$$

   Here we have $E(X)=2$, $t=1$ and $B=10$. The right hand side is $1/9 \geq 1/10$.

   (c) $V(X+Y) = 9+V(Y)$ is **TRUE** by calculation: $V(X+Y) = E[(X+Y)^2] - (E(X+Y))^2$ and this is $V(X)+V(Y)+2[E(XY)-E(X)E(Y)]]$.

   (d) $P(X \geq 8) \leq 1/4$ is **FALSE**. The temptation is to invoke Markov's inequality, but we are NOT GIVEN the necessary fact that X is a non-negative random variable. If X is 4 or -1, each with probability 1/2, the required data for X are satisfied but not the stated probability inequality.

   (e) $P(X \geq 5) \leq 2/5$ is **TRUE** by Tchebycheff's inequality

   (f) **FALSE** - X and Y are NOT necessarily independent, just because $E(XY)=E(X)E(Y)$. I will try to put in a simple counterexample later.

2. (a)

   $$2\frac{(k!)(k!)}{(2k)!}$$

   (b)

   $$\frac{\binom{2k}{k} k!}{(2k)!}$$

   There are $\binom{2k}{k}$ ways to determine the locations of the odds. Given the locations, there is only one ordering for the odds - they have to increase. There are $k!$ orderings of the evens in the remaining locations.

   For the algorithm, generate $(\pi_1, \ldots, \pi_{2k})$ a random permutation. Initialize another array A of size $2k$ to zero. The first k entries in π point to where the evens go, so set $A[\pi_j] \leftarrow 2j$, $j = 1, \ldots, k$. The last k entries of the permutation point to where the odds go. So just scan the A array and set $2j-1$ in place of the j^{th} zero in A, $j = 1, \ldots, k$. The alg is linear.

(c)

$$\frac{((k-1)!)^2}{(2k)!}$$

(d)

$$\frac{\binom{2k}{k}(k-1)!}{(2k)!}$$

$P_D(C) = 0 \neq P(C)$, so they are NOT independent.

3. I gave 5 pts here only if some info was given
4. Randomly sample.
5. (a) G is a cycle and a "tail". The min-cut is the tail separating the vertex of degree one from the other $n-1$ vertices.

   (b) $2/n$.