CS206 **Practice Problems II** **April 13, 2012**

(these are ALL practice problems, not to be handed in

1. Your computer program has 3 bugs. Each time you try some corrections there is a probability of $\mathcal{P} = 1/3$ that you remove some bug. Only one bug can be removed at a time.
   (a) What is the probability that you remove one bug in less than 10 tries?
   (b) What is the probability that you remove ALL bugs in less than 10 tries?
   (c) Repeat the above given that you remove the first bug on the second try.
   (d) What is the expected number of tries to remove two bugs.
   (e) What is the variance of the number of tries needed to remove ALL bugs?

2. Toss a fair coin twice and let X count the number of heads, Y, the number of tails, and let $Z = (X - Y)^2$.
   (a) Find $E(X), E(Y), E(Z)$ and $V(X), V(Y), V(Z)$. Now compute the variance of $X + Y$; of $X + Z$. What does this say about the independence of these random variables?
   (b) As above, now find the generating functions ϕ_X, ϕ_Y, and ϕ_Z, Use them to compute $E(X), E(Y)$, and $E(Z)$.
   (c) Use generating functions to test the independence of X and Z.
   (d) Repeat (a) if you toss the coin *three* times.

3. Repeat the above for the two dice experiment where X is the score of the red die, Y the score of the blue die, and $Z = X + Y$ is the sum.

4. This question is aimed at the use of Tchebycheffs' inequality to check some assigned probability for a random experiment. In each part there are three aspects of the inequality, the number n of observations, the (absolute) difference t between the actual and expected number of observations, and the confidence you have in asserting that this difference is "large". Each part will "give", or imply two of these quantities, and ask you for the third.
   (a) In 1000 tosses of a coin you observe 300 heads. What does this say about the fairness of the coin?
   (b) Suppose that in n tosses of a coin you got $n/3$ Tails. What should n be to make you 95 percent confident that the coin is NOT fair?
   (c) You toss a die $n = 6000$ times. What is the fewest number of "threes" that would convince you with 99 percent confidence that the die is biased in favor of "three"?

5. The experiment $\mathcal{E}$ is to take a required computer science course. The outcomes are the grades $\{A, B^+, B, C^+, C, D, F\}$. Suppose $\mathcal{E}$ is repeated 5 times, once for each of CS111, CS112, CS205, CS206, and CS344. Assume the grades are equally likely.
   (a) Let X count the number of distinct grades you receive in the five courses. Compute the frequency function f_X and use it to compute $E(X)$. [we did this in an earlier HW]
   (b) Find the generating function ϕ_X and use it to compute $E(X)$ and $V(X)$.

OVER

(These are practice problems on the ballot theorem and binary trees. (*) is more challenging)

1. How many rooted binary trees with 7 nodes are there? How many of them have three nodes in the left subtree? How many of them have height 3?
2. (*) You choose a 7-node tree at random (equally likely probability). What is the probability its height is 3 given there are three nodes in the left subtree?
3. 75 people have \$5 bills and 25 people have \$10 bills, and they are in line at a ticket counter which has no money and which charges 5 dollars for admission. If a \$10 bill is presented and there is no change, *the line stops.* What is the probability that the ticket seller always (after the first person in line, that is) has at least one \$5 bill for change? (so how many paths from $(0, 0)$ to $(100, 50)$ are positive?)
4. As above, what is the probability the ticket seller was always able to make change. (that is, how many paths from $(0, 0)$ to $(100, 50)$ never go negative?)
5. (*) In 4), what is the probability that the ticket seller was always able to make change if he is now allowed to use his own, single \$5 dollar bill, if needed? (that is, how many paths from $(0, 0)$ to $(100, 50)$ never go below $y = -1$?
6. Repeat questions 3-5 if now, 60 people have \$5 bills and 20 people have \$10 bills.
7. In an election where A gets 100 votes and B gets 50 votes, what is the probability that A will be leading throughout the counting of the ballots?
8. In 7, what is the probability that A never trails during the counting?
9. (*) In 7, what is the probability that A never trails by more than 1 vote throughout the counting?
10. How many random walk paths start at $(0, 0)$ and end at $(12, 2)$?
11. How many of the above paths are positive? non-negative?
12. (*) Of the paths in 10), how many pass through $(6, 0)$
13. (trick question) How many random walk paths start at $(0, 0)$ and end at $(12, 5)$?