(These are practice problems, NOT TO HAND IN. I may add to them)

1. Two fair dice are rolled. What is the probability that the number showing on one of them will be twice the number showing on the other? **ANS:** \((1,2), (2,1),(2,4), (4,2), (3,6), (6,3)\) are the outcomes, so the prob is \(1/6\).

2. An urn has 5 white, 4 black, and 3 red chips. Four chips are drawn at random, without replacement. What is the sample space? What is the probability that you got W.R.W.B (this is event \(A\))? That you got at least two whites (this is event \(B\))? What is \(P_B(A)\)? **ANS:** Imagine the chips are also numbered 1, \ldots, 12 so the experiment can be viewed as sampling 4 times without replacement and the sample space has size \(|S| = (12)_4 = 11,880\). There are \(5 \cdot 3 \cdot 4 \cdot 4\) outcomes in \(A\) so \(|A| = 240\).

3. Urn I has three red, two black, and five white chips. Urn II has two red, four black, and three white. One chip is drawn at random from each urn. What is the probability that both chips are the same color? Given that they are the same color, what is the probability of White? **ANS:** The sample space has 90 outcomes: 10 choices for the chip from Urn 1 and for each, 9 choices for the chip from Urn 2, and all possibilities are equally likely. The chips are the same color in 6 different ways (red/red) + 8 ways (black/black) + 15 ways (white/white) for a total of 29 so the probability is 29/90. If \(A\) is \{both chips same color\} and \(B\) is the event \{both chips are white\}, \(P_A(B) = P(A \cap B)/P(A) = |A \cap B|/|A| = 15/29\) so (although we were not asked, the events are NOT independent.

4. How many ways can you choose four different members to serve as president, vice-president, and treasurer and secretary from club with 25 members? If the club is 12 men and 13 women, how many of these choices have no male officers? No male president? **ANS:** \((25)_4\) choices. \((13)_4\) have no male officers; \((13)(24)_3\) have no male president.

5. A liquor store owner will cash checks up to 50 dollars, but is wary about customers wearing sunglasses. Fifty percent of checks written by customers wearing sunglasses bounce while only two percent of the checks written by persons not wearing sunglasses bounce. Half the customers wear sunglasses. If a certain check bounces, what is the probability it had been written by a customer with sunglasses? **ANS:** easy Bayes rule application.

6. How many even numbers in \([100,999]\) have distinct digits? How many palindromes (numbers that are the same when you write them backwards) are in this range? How about the range \([1000,9999]\)? **ANS:** I counted 328. To be even, a number must end with 0, 2, 4, 6, 8 as its right-most digit. If that digit is 0, there are nine distinct choices for the tens digit and for each, eight other choices for the (different) hundreds digit. But if the units digit is NOT 0, there are four other possibilities (2, 4, 6, 8) and for each, eight choices for the hundreds digit (not 0 and not the chosen non-zero, even, units digit). Adding these possibilities we get \(9 \cdot 8 + 4 \cdot 8 \cdot 8 = 328\). There are 90 palindromes (9 first digits and 10 middle digits).

7. (***) Find the probability that in a bridge deal, NO player gets 13 cards of the same suit. **ANS:** The sample space has size \(|S| = \binom{52}{13} \binom{13}{13} \binom{13}{13} \binom{13}{13} = 52!/(13!)^4\). We compute the size of the complement \(A^c = A_1 \cup A_2 \cup A_3 \cup A_4\) where \(A_i\) is the event that player \(i\) got all cards of one
suit (1 is North, 2 is East, etc.). By the inclusion/exclusion formula 
\[ P(A^c) = S_1 - S_2 + S_3 - S_4 \]
where
\[ S_1 = P(A_1) + \cdots + P(A_4) = 4 \cdot P(A_1) \text{[WHY?]}, \]
\[ S_2 = P(A_1 \cap A_2) + P(A_1 \cap A_3) + \cdots + P(A_3 \cap A_4) = 6 \cdot P(A_1 \cap A_2) \]
\[ S_3 = 4 \cdot P(A_1 \cap A_2 \cap A_3) \]
and
\[ S_4 = P(A_1 \cap \cdots \cap A_4) = S_3/4 \text{[WHY?]}. \]

\[ S_1 = 4 \cdot P(A_1) = 4 \cdot [4 \cdot \binom{39}{13}/|S|] \] because there are four suits player 1 could be given, and for each, \( \binom{39}{13} \) choices for player 2’s hand, and for each of those hands, \( \binom{26}{13} \) possible hands for player 3; player 4 gets the last 13 cards.

\[ S_2 = 6 \cdot 4 \cdot 3 \cdot \binom{26}{13}/|S| \] (6 pairs of players to each get hands of a single suit, 4 possible suits for the first of them and for each choice, 3 suits for the second, and finally \( \binom{26}{13} \) ways to make the remaining two hands.

Using the same reasoning, \( S_3 = 4 \cdot 4 \cdot 3 \cdot 2/|S| \) and \( S_4 = 4 \cdot 3 \cdot 2/|S| \).

8. As above, find the probability that exactly two players each got dealt their 13 cards from a single suit. [CAREFUL!]