

## Old Midterm, June 21, 1994 - Some Answers

## 1. Question 1:

- (a) I will do this as an UNordered sample of size 6 from a set of 30 club members (though it could be done as an ordered sample as well).  $S$  is thus the collection of all distinct groups of 6 members and  $|S| = \binom{30}{6}$ .
- (b)  $|A| = 25$  and  $P(A) = |A|/|S|$ .
- (c)  $|B| = \binom{22}{6}$ .
- (d)  $P(C) = 1 - P(C^c)$  and  $|C^c| = \binom{23}{6}$ .
- (e) They are NOT independent because

$$P_B(C^c) \neq P(C^c).$$

The left side is  $P(B \cap C^c)/P(B) = |B \cap C^c|/|B| = \binom{15}{6}/\binom{22}{6}$ . The right side is  $\binom{23}{6}/\binom{30}{6}$ , and its easy to see these expressions are different (left side is smaller).

## 2. Question 2

- (a)  $S = \{(t_1, \dots, t_5) : t_i \in \{1, 2, 3, 4, 5, 6\}\}$  is what occurred on the  $i^{\text{th}}$  toss}.  $|S| = 6^5$ . I will use equally likely probability because the die is "fair".
- (b)  $|A| = (6)_5$  and  $P(A) = |A|/|S| = 5!/6^4$ .
- (c)  $|B| = 3^5$ ; each of the 5 tosses has 3 possible (even) outcomes.
- (d)  $|C| = \binom{5}{3} \cdot 6 \cdot 5$ : 6 choices for the value that appears 3 times; 5 choices for the (different) value that appears twice;  $\binom{5}{3}$  ways to choose which three tosses will have the same outcome.
- (e)  $|D| = \binom{7}{2}6!$  and  $|S| = 6^7$ .

## 3. Question 3.

- (a)  $4!/6! = 1/30$ .
- (b)  $\binom{6}{4}/6!$ ; once you know which 4 get their own hats, the *other* pair must swap hats.
- (c)  $\binom{6}{3} \cdot 2/6!$ ; once you know which 3 get their own hats, the other 3 must be deranged (2 of the 6 permutations of their hats).